MCV 4U Date _________

Calculus – Test 3 Review

1. Find the limits of the following functions:

a) $\lim_{x\to-\infty}\frac{1}{3x-2}$ b) $\lim_{x\to 2}\frac{1}{x^2-4}$ c) $\lim_{x\to\infty}\frac{5x-6}{2x+1}$ d) $\lim_{x\to 2}\frac{x-2}{x^2-4}$

2. From first principles, find the derivative function, $f'(x)$, for the following functions:

a) $f(x)=x^2-3x+2$ b) $f(x)=\frac{2x+1}{3x-4}$

3. Find the derivative function for each of the following: a) $y=-\frac{1}{3}x\ +5x^2+6$ b) $y=5^x$ c) $y=\frac{1}{3}\cos x$

d) $y=5\sin x$ e) $y=-3e^x$ f) $y=(4x-3)^2$

4. Find the derivative of $y=x^3$ from first principles.

5. A particle moves along a horizontal line so that at time t seconds its position (in meters) is $y=20\sin t$ from a reference point O on the line. For example, at time $\frac{\pi}{2}=\frac{3.14}{2}=1.57$ seconds the object is 20 meters to the right of the reference point O.

a) Find the particles position with respect to O at 2 seconds?

b) Find the rate of change from 4 to 5 seconds?

c) Find the rate of change at 5 seconds?

6. Find the derivative function for each of the following: (**DO NOT SIMPLIFY)**

a) $y=(3x-4)^5$ b) $y=2^{2x^2-4x}$ d) $y=e^{\sin x}$ e) $y=\frac{1}{4}x^3\sin(2x)$

g) $f(x)=(2x^2-7)^5\sqrt{3x-5}$ h) $g(x)=\frac{(3-x^3)}{(2x+1)}$ c) $y=3\cos(x^2-2x+6)$ f) $y=\sqrt{x^3-\frac{1}{2}x}$

i) $y=\frac{-5}{(5x^2-2x+1)^4}$ j) $f(x)=\frac{\sin^3(2x)}{(2x^3-3x)}$

7. Find the derivative function for each of the following: (**SIMPLIFY)**

a) $y=(5x+1)^4(2x-3)^2$ b) $y=\frac{(3x-1)}{\sqrt{3x}}$

8. A pendulum is swinging back and forth. The angle θ that the pendulum's string makes with a vertical line can be modeled by the function $\theta=0.2\cos 4t$ where θ is in radians and t is in seconds.

a) Find the angle the string makes with the vertical at 2 seconds?

b) Find the rate of change of the angle at any time t.

c) Find the rate of change of the angle at $t=1$ second.

9. A motorboat coasts toward a dock with its engine off. Its distance d, in meters, from the dock t seconds after the engine is turned off is given by $d(t)=\frac{10(6-t)}{t+3},\ 0\le t\le 6$.

a) How far is the boat from the dock at the instant the engine is turned off?

b) What is the rate of change of the distance with respect to time (i.e. velocity) for the boat when it bumps into the dock?

10. The definition of the derivative of a function is given by $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. Using the definition of the derivative and algebraic techniques, attempt to find the derivative of:

a) $f(x) = \frac{x+1}{3x-4}$

b) $f(x) = (x-2)^3$

11. For the following function:

i) determine all local maximum and minimum points.

ii) specify the interval(s) over which the function is increasing.

$f(x) = (x-2)^3(x-6)$

12. For the following function:

i) determine the coordinates of all points of inflection.

ii) specify the interval(s) over which the function is concave down.

$f(x) = \frac{x^2}{x+3}$

a) the interval(s) for which f is increasing.

b) value(s) for x at which f'' changes sign

c) value(s) for x at which the function is undefined

d) the interval(s) over which the function is concave downward

e) the coordinates of any local minimum point

13. A particle moves on the x axis with this relationship between position and time, velocity and time and acceleration and time ($t \in \Re$):

$s(t) = t^4 - 9t^3 + 27t^2 - 27t = t(t-3)^3$, $v(t) = 4t^3 - 27t^2 + 54t - 27 = (4t-3)(t-3)^2$, $a(t) = 12t^2 - 54t + 54 = 6(2t-3)(t-3)$

a) Describe the motion of the particle at $t = 0$
b) When is the particle to the left of the origin?
c) When is the particle moving right?
d) What is the average acceleration of the particle between $t = 0$ and $t = 1$ seconds?
e) When does the particle reverse direction?
f) Find the total distance traveled from $t = 0$ to $t = 2$ seconds.
g) Determine the time interval(s) during which the particle is moving away from the origin.
h) Determine the time interval(s) during which the particle is slowing down.
i) Determine the time interval(s) during which the particle is moving away from the origin and slowing down.

14. A cylindrical container is to hold a volume of $300\ cm^3$. The side of the container is made of plastic which costs \$0.04 per cm^2 and the top and bottom are made of metal which costs \$0.07 per cm^2. Find the radius and height which will minimize the cost of manufacturing this container. Find the minimum cost of manufacturing the container.

15. Given $y = \sqrt{x}$, find $\frac{d^2y}{dx^2}$

16. A closed right circular cylinder is such that the sum of its height and the circumference of its base is 10 m. Find the maximum volume of the cylinder.

17. Find $\frac{dy}{dx}$ for: a) $y=\frac{e^x}{x^2-1}$ b) $y=3^x 2e^{-3x^2}$ c) $y=5x(4x-5)^{10}$ d) $y=4xe^{-2x}$ e) $f(x)=\frac{x^3-2x}{x+5}$

18. The position in kilometres of a particle at t hours is given by $d(t)=t^3-12t^2+34t+75$, where $t\geq 0$.

a) What is the initial position of the particle?
b) What is the particle's velocity at t = 6?
c) What is the particle's velocity at t = 7?
d) Explain the meaning of the sign change in parts b) and c).
e) When is the particle stationary?

19. Find the domain, intercepts, asymptotes, interval(s) of increase and decrease, maxima and minima, interval(s) of concavity, and point(s) of inflection for each of the following. Sketch the graph.

$f(x)=\frac{-3x}{\sqrt{x-1}}$ $\quad f(x)=xe^x$ $\quad f(x)=\frac{10-10x}{(x-4)^2}$

20. Analyse each of the following functions and use your analysis to sketch the graph of the function.

a. $n(x)=xe^{-3x}$, $\quad n'(x)=(-3x+1)e^{-3x}$, $\quad n''(x)=3(3x-2)e^{-3x}$

h) $l(x)=\ln(x^2-3)$, $\quad l'(x)=\frac{2x}{x^2-3}$, $\quad l''(x)=\frac{-6}{(x^2-3)^2}$

21. The position of a particle is given by $s(t)=4t^3-20t$ where *t* is measured in seconds and position *s* is measured in metres. Find the average acceleration from 1 s to 5 s.

22. Given that $f'(x)=3x+e^{-x}$, where is *f(x)* concave down?

23. Find the slope of the secant of $y=2x^2-3x$ that passes through the points where $x=-3$ and $x=1$.

24. The concentration of medicine in a patient's bloodstream is given by $C(t)=\frac{0.4t}{(0.3t+2)^3}$, $t\geq 0$, where *C* is measured in milligrams per cubic centimetre and *t* is the time in hours after the medicine was taken. How long does it take for the medicine to reach its greatest concentration?

25. A 3 m length of wire has to be bent into the form of a rectangle with an external circular loop at one corner such that the rectangle is to have one side length double the other. Find the dimensions of the circle and the rectangle so that the total area is minimised.

26. The atmospheric pressure, y, varies with the altitude, *x* kilometres, above the Earth. For altitudes up to 10 km, the pressure in millimeters of mercury (mm Hg) is given by $y=760e^{-0.125x}$.

a) What is the atmospheric pressure 5 km above the Earth?
b) Determine the rate at which the atmospheric pressure is changing when you are 9 km above the Earth.

27. Find the radius and height of a right-circular cylinder of largest volume that can be inscribed in a right-circular cone with radius 12cm and height 20 cm.

28. A pipe is to be carried around a corner from a hallway that is 4 m wide into a hallway that is 4 m wide. What is the maximum length of the pipe?