MCV 4U

Lines Review

1. Convert each of the following equations to the requested form.

a) $\frac{x+8}{4}=\frac{y-9}{-2}=\frac{5-z}{-1}$ to vector form.

b) $\vec{r}=(2,-1)+t(-2,7)$ to scalar form.

c) $\begin{cases} x=-3+4t \\ y=-t \\ z=2-2t \end{cases}$ to symmetric form.

d) $(x,y,z)=(-1,4,9)+k(5,-2,2)$ to parametric form.

e) $3x-5y-15=0$ to vector form.

2. Find a normal vector for a line which is:
   a) parallel to $5x-6y-15=0$.
   b) perpendicular to $(x,y)=(-1,0)+k(3,-8)$

3. Find the symmetric equation of the line through $P(-3,5)$ with slope $^{-9}/_{4}$.

4. Find the scalar equation of the line with direction vector $\vec{d}=(-3,1)$ passing through $P(-4,7)$.

5. Find the requested version of the line described in each of the following:

   a) Vector equation of the line parallel to $4x-3y+12=0$ and passing through the y intercept of $\begin{cases} x=-4+2k \\ y=3-5k \end{cases}$.
   b) Scalar equation of the line perpendicular to $2x-3y+8=0$ passing through $P(2,-3)$.
   c) Parametric equations of the line through $A(-3,2,5)$ and is perpendicular to both ℓ_1 *and* ℓ_2 where

$$\ell_1: \frac{x-4}{3}=y-2=\frac{z-3}{-2} \qquad \ell_2: (x,y,z)=(-1,1,5)+k(-1,-2,3)$$

6. Find the intersection of the following lines. Then classify them as an inconsistent or consistent, dependent or independent system.

a) $\ell_1: (x,y)=(-1,3)+t(-2,4)$, $\ell_2: 2x+y-1=0$

b) $\ell_1: (x,y,z)=(-1,3,5)+t(1,-2,6)$ and $\ell_2: \begin{cases} x=-13+3k \\ y=-8+k \\ z=-1+3k \end{cases}$

c) $\ell_1: \frac{x-2}{1}=\frac{y-1}{-1}=\frac{z}{1}$, $\ell_2: \frac{x-3}{2}=\frac{y}{3}=\frac{1-z}{1}$

d) $\ell_1: \begin{cases} x=1+t \\ y=1+2t \\ z=1-3t \end{cases}$ and $\ell_2: \begin{cases} x=3-2u \\ y=5-4u \\ z=-5+6u \end{cases}$

7. Show that the lines $\vec{r}=(4,7,-1)+t(4,8,-4)$ and $\vec{r}=(1,5,4)+u(-1,2,3)$ intersect at right angles and find the point of intersection.

8. Find the distance between:

a) The point (3, 7) and the line $2x-3y=7$

b) lines: $\ell_1 : 5x-2y+25=0$, $\ell_2 : 5x-2y-5=0$

c) the point $A(-6,5,-3)$ and the line $(x,y,z)=(6,1,3)+t(5,-3,3)$

d) the lines $\ell_1 : \vec{r}=(1,6,-2)+t(1,-2,5)$, $\ell_2 : \vec{r}=(3,-4,-9)+k(-2,7,1)$

e) lines: $\ell_1 : \frac{x-1}{2}=\frac{y+4}{1}, z=1$ and $\ell_2 : \begin{cases} x=4t \\ y=1+2t \\ z=6 \end{cases}$

9. Create a system of two equations in 3-space so that they intersect at the point (5,5,5) and are at an angle of $55°$ degrees to each other. **Justify**.

SOLUTIONS:

1. a) $(x,y,z)=(-8,9,5)+k(4,-2,1), k\varepsilon R$ b) 7x + 2y – 12 = 0 c) $\frac{x+3}{4}=\frac{y}{-1}=\frac{z-2}{-2}$ d) $x=-1+5k, y=4-2k, z=9+2k$ e) $(x,y,z)=(0.-3)+k(5,3), k\varepsilon R$	5. a) $(x,y)=(0,-7)+k(3,4)$ b) 3x + 2y = 0 c) x = - 3 – t, y = 2 – 7t, z = 5 – 5t
2. a) (5, - 6) b) (3, - 8)	6. a) $(t, 1-2t), \quad t \in \Re$, b) { system is inconsistent }, c) { (3, 0, 1)}, d) { (a, 2a - 1, 4 - 3a)}
3. $\frac{x+3}{4}=\frac{y-5}{-9}$	7. { (2, 3, 1)}
4. x + 3y – 17 = 0	8. a) { 6.10 units}, b) { 5.57 units}, c) { 2.76 units}, d) { 0.39 units}, e) { 7.01 units}