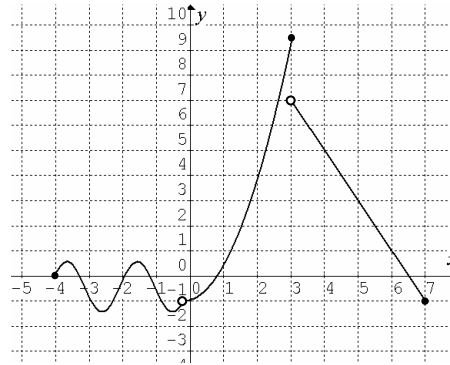


MCV4U Review for Calculus Test no. 1

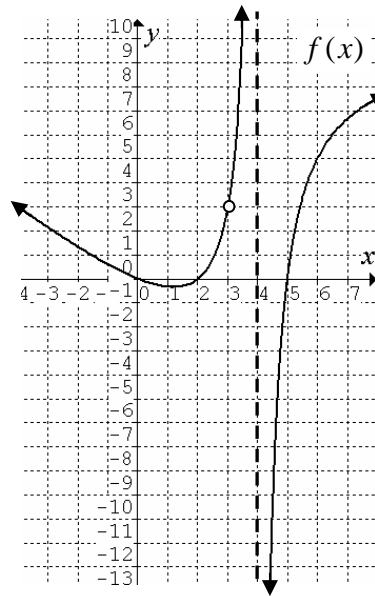
1) Given the graph of the function $f(x)$ shown on the right, estimate the point(s) / interval(s) that satisfies each set of criteria for $-4 \leq x \leq 7$.

- a) function is increasing
- b) function is concave down
- c) instantaneous rate of change is constant
- d) instantaneous rate of change is increasing
- e) first differences are positive
- f) second differences are negative
- g) removable discontinuity
- h) jump discontinuity
- i) where the instantaneous rate of change equals the average rate of change for $-4 \leq x \leq -2$



2) Use the given graph of $f(x)$ to state the value of the limit, if it exists

- a) $\lim_{x \rightarrow 6} f(x)$
- b) $\lim_{x \rightarrow 4^-} f(x)$
- c) $\lim_{x \rightarrow 4^+} f(x)$
- d) $\lim_{x \rightarrow 4} f(x)$
- e) $\lim_{x \rightarrow -\infty} f(x)$
- f) $\lim_{x \rightarrow \infty} f(x)$
- g) $\lim_{x \rightarrow 3} f(x)$



3) a) Sketch the graph of the function $f(x) = \begin{cases} (x+1)^2, & x < -1 \\ x, & -1 \leq x \leq 1 \\ 2x - x^2, & x > 1 \end{cases}$

b) Determine if the function is continuous. If it is not continuous, state why not and identify the type of discontinuity.

4) Find the following limits, if they exist.

- a) $\lim_{x \rightarrow 1} (3x - 7)$
- b) $\lim_{x \rightarrow 4} \frac{x^2 + 2x - 3}{x^2 + 2}$
- c) $\lim_{x \rightarrow -4} \sqrt{x^4 + 2x^2}$
- d) $\lim_{x \rightarrow 0} \frac{(x-2)^2 - 4}{x}$
- e) $\lim_{x \rightarrow 3} \frac{1}{(x-3)^2}$
- f) $\lim_{x \rightarrow -2} \frac{x^2 - x - 2}{x^2 + 3x + 2}$
- g) $\lim_{x \rightarrow 1} \frac{x-1}{x^4 - 1}$
- h) $\lim_{x \rightarrow 0} f(x)$, given $f(x) = \begin{cases} -1, & x < 0 \\ x+1, & x \geq 0 \end{cases}$
- i) $\lim_{x \rightarrow -2} f(x)$ and $\lim_{x \rightarrow 2} f(x)$ given $f(x) = \begin{cases} -1, & x \leq -2 \\ 0.5x, & -2 < x < 2 \\ 1, & x \geq 2 \end{cases}$
- j) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$
- k) $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$
- l) $\lim_{x \rightarrow \infty} \frac{1}{x}$
- m) $\lim_{x \rightarrow \infty} (5x)$
- n) $\lim_{x \rightarrow \infty} \frac{x^2 + 1}{2x^2 - 1}$
- o) $\lim_{x \rightarrow \infty} \frac{x^3 - x^2 + 4x + 7}{x^2 - 4}$

- 5) Given the function $f(x) = x^2 + 4x$, $-7 \leq x \leq 4$
- Create a table of values.
 - Determine the first and second finite differences.
 - Use the finite differences to determine the interval(s) over which the function is increasing, decreasing, concave up, and/or concave down.
 - Sketch the graph of the function using the information from the finite differences.
 - Sketch the graph of the function using your knowledge of quadratic functions and compare to the previous graph.

6) Sketch the graph of a polynomial that satisfies the following conditions:

- $f(x) \geq 0$, when $x \leq -4$, when $x = 1$ and when $x \geq 3$
- first differences are positive when $-2.6 < x < 1$ and when $2.4 < x$
- second differences are positive when $x < -1.2$ and when $1.7 < x$.

Are there other possible graphs?

7) a) Determine the expression for the slope of any secant line passing through $(2, -4)$ on the graph of

$$g(x) = x^2 - 4x.$$

b) Use your answer from a) to determine the average rate of change of the secant line passing through

$$(2, -4) \text{ and } (5, 5). \text{ Verify your answer using } \frac{\Delta f(x)}{\Delta x}.$$

c) Determine the instantaneous rate of change of $g(x) = x^2 - 4x$ at $x = 2$.

8) Determine the instantaneous rate of change of $f(x) = \frac{x^2 - 4x + 1}{x - 1}$ at $x = 3$.

9) Find the derivative of $y = -3x^2 - 2x - 4$ from first principles (using the IROC function and algebraic manipulation).

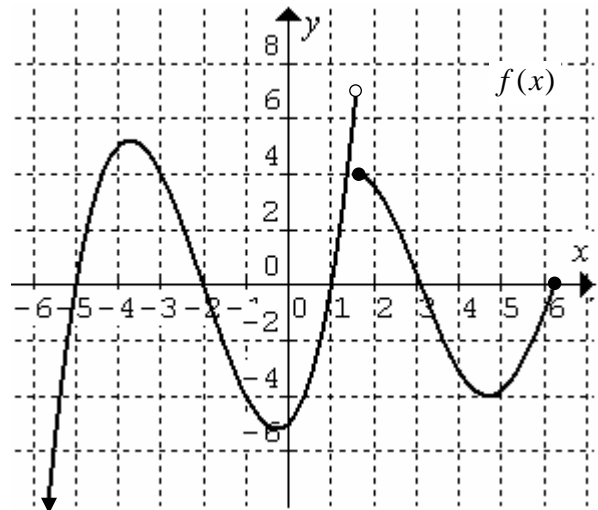
10) Find $f'(x)$ for $f(x) = 2x^3 - x^2 + 5x - 6$ by first principles.

11) Find $f'(x)$ for $f(x) = \frac{3}{x-1}$ by first principles.

12) Find $f'(x)$ for $f(x) = \sqrt{2x-3}$ by first principles.

13) Given the graph of the (piecewise) function $f(x)$ shown on the right, estimate the point(s) / interval(s) that satisfies each set of criteria for $x \leq 2\pi$.

- the function is positive
- the function is decreasing
- the derivative has a zero
- the derivative is positive
- the derivative is increasing
- the derivative is negative and decreasing
- the derivative has a turning point
- the derivative has a maximum



14) Given $f(x)$ is a polynomial function of degree 6, what can you tell about the derivative of the function?

15) Go back through all of the activities we have done and write down your conclusions and notes.