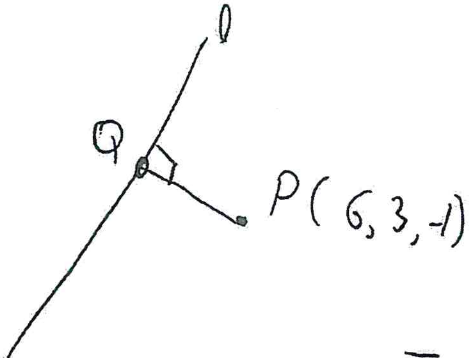
Challenge Set #4

1)



let Q be $(3+4t, -2+3t, -2t)$

then $\overline{PQ} = (-3+4t, -5+3t, 1-2t)$

$$\overline{PQ} \perp \bar{d}$$

$$\therefore (-3+4t, -5+3t, 1-2t) \cdot (4, 3, -2) = 0$$

$$-12 + 16t - 15 + 9t - 2 + 4t = 0$$

$$29t = 29$$

$$t = 1$$

$\therefore$ Q is the pt $(7, 1, -2)$

To verify, I will compare $|\overline{PQ}|$ to the distance between P and l

$$|\overline{PQ}| = |(1, -2, -1)|$$

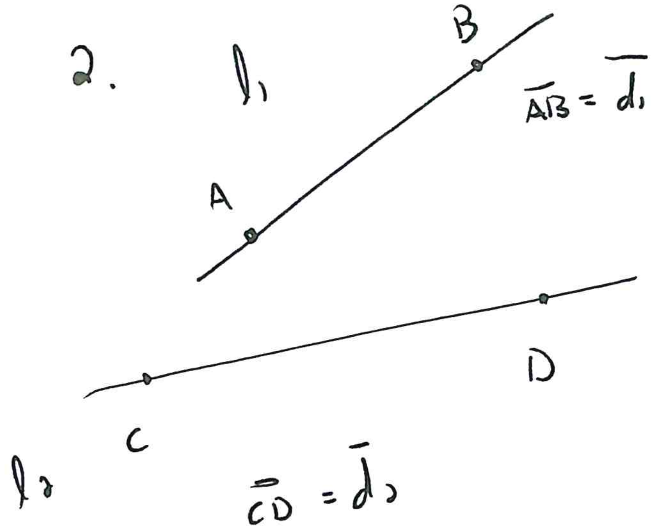
$$= \sqrt{6}$$

$$\text{dist} = \frac{|\overline{PP_0} \times \bar{d}|}{|\bar{d}|}$$

$$= \frac{|(3, 5, -1) \times (4, 3, -2)|}{\sqrt{29}}$$

$$= \frac{|(7, 2, -11)|}{\sqrt{29}} = \frac{\sqrt{174}}{\sqrt{29}} = \sqrt{6}$$

2.



$$\text{dist} = \frac{|(\bar{d}_1 \times \bar{d}_2) \cdot (\overline{AD})|}{|\bar{d}_1 \times \bar{d}_2|}$$

$$= \frac{(-3,-3,-4) \times (0,1,2) \cdot (5,-5,3)}{|\bar{d}_1 \times \bar{d}_2|}$$

$$= \frac{(-2,6,-3) \cdot (5,-5,3)}{|(-2,6,-3)|}$$

$$= \frac{|-49|}{7}$$

$$= 7$$

$l_1 : (x,y,z) = (-2,2,-6) + k(-3,-3,-4)$
$l_2 : (x,y,z) = (3,-4,-5) + t(0,1,2)$

Let P, Q be the points. Then $\overline{PQ} \perp \bar{d}_1$,

$\overrightarrow{PQ} \perp \bar{d}_2$.

$\overline{PQ} = (-5-3k,\ 6-3k-t,\ -1-4k-2t)$

$\bar{PQ} \cdot \bar{d}_1 = 0$

$\therefore\ 15 + 9k - 18 + 9k + 3t + 4 + 16k + 8t = 0$

$34k + 11t = -1$

$\bar{PQ} \cdot \bar{d}_2 = 0$

$\therefore\ 6 - 3k - t - 2 - 8k - 4t = 0$

$-11k - 5t = -4 \quad \Rightarrow \quad 11k + 5t = 4$

We need to solve

$$\begin{aligned} 34k + 11t &= -1 \\ 11k + 5t &= 4 \end{aligned} \quad \Rightarrow \quad \begin{aligned} k &= -1 \\ t &= 3 \end{aligned}$$

$\therefore\ P = (-2, 2, -6) - 1(-3, -3, -4)$

$= (1, 5, -2)$

$Q = (3, -4, -5) + 3(0, 1, 2)$

$= (3, -1, 1)$

$\bar{PQ} = (2, -6, 3)$

$|\bar{PQ}| = 7$

3.

Analyze direction vectors to confirm non-scalar multiples

1) Solve for POI, if it exists

2) Determine distance between $\Rightarrow$ either zero or not zero.

3) Use the two direction vectors plus a 3rd vector made between a point on each line, test the 3 vectors for coplanarity