Challenge Set #2

4. answers vary, let one component be zero and find neg. recip. of remaining 2.

$\bar{u}_{\perp} = (0, 4, -5)$

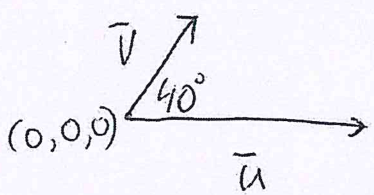
$(0,4,-5) \cdot (-2,5,4)$

$= 0 + 20 - 20$

$= 0$

5. Let $\bar{u}, \bar{v}$ be the defining vectors

answers vary



Define $\bar{u}$ to be on x-axis

suppose $\bar{u} = (8,0,0)$

Let $\bar{v} = (x, 1, 1)$ ⇒ any vector not collinear with $\bar{u}$

$\therefore \cos 40 = \dfrac{\bar{u} \cdot \bar{v}}{|\bar{u}|\,|\bar{v}|}$

$0.766 \doteq \dfrac{8x}{8\sqrt{x^2+2}}$

$0.766 \doteq \dfrac{x}{\sqrt{x^2+2}}$

$0.766\sqrt{x^2+2} \doteq x$

$0.587(x^2+2) \doteq x^2$

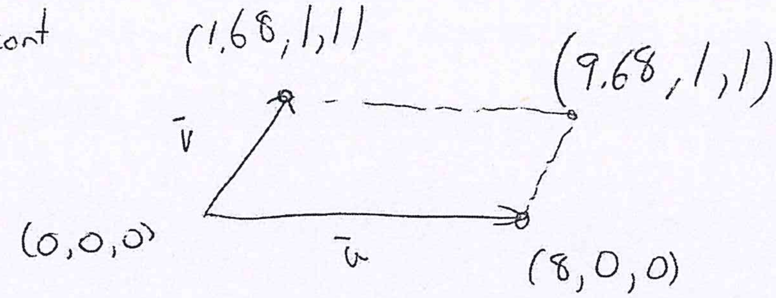
$0.587x^2 + 1.17 \doteq x^2$

$x^2 \doteq 2.833$

$x \doteq \pm 1.68$

$\therefore \bar{v} = (1.68, 1, 1)$

5. cont



6. $\bar{a} \perp \bar{b}$ Show $|\bar{a}|^2 + |\bar{b}|^2 = |\bar{a}+\bar{b}|^2$

$$
\begin{aligned}
RS &= |\bar{a}+\bar{b}|^2 \\
&= (\bar{a}+\bar{b}) \cdot (\bar{a}+\bar{b}) \\
&= \bar{a} \cdot \bar{a} + 2\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{b} \\
&= |\bar{a}|^2 + 2(\bar{a} \cdot \bar{b}) + |\bar{b}|^2
\end{aligned}
$$

But $\bar{a} \cdot \bar{b} = 0$

$$
\begin{aligned}
\therefore RS &= |\bar{a}|^2 + |\bar{b}|^2 \\
&= LS
\end{aligned}
$$

Pythagoreon Thm!

7. $\bar{a} \cdot \bar{b} \neq 0$

$\bar{c} = \bar{a} - \bar{b}$

$$
\begin{aligned}
|\bar{c}|^2 &= \bar{c} \cdot c \\
&= (\bar{a}-\bar{b}) \cdot (\bar{a}-\bar{b}) \\
&= \bar{a} \cdot \bar{a} - 2\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{b} \\
&= |\bar{a}|^2 + |\bar{b}|^2 - 2\bar{a} \cdot \bar{b} \\
&= |\bar{a}| + |\bar{b}| - 2|\bar{a}||\bar{b}|\cos\theta
\end{aligned}
$$

Cosine Law!

8. $(\bar{a}+\bar{b})\cdot(\bar{a}-\bar{b})=0$

$\bar{a}\cdot\bar{a}-\bar{a}\cdot\bar{b}+\bar{a}\cdot\bar{b}-\bar{b}\cdot\bar{b}=0$

$|\bar{a}|^2-|\bar{b}|^2=0$

$(|\bar{a}|-|\bar{b}|)(|\bar{a}|+|\bar{b}|)=0$

$\therefore |\bar{a}|=|\bar{b}|$

$\therefore$ This is true if $\bar{a},\bar{b}$ are vectors with the same magnitude. Direction is not important

9. $|\bar{a}\cdot\bar{b}|=|\bar{a}||\bar{b}|\cos\theta$

Since $\cos\theta$ is a number between -1 and 1

ie $|\cos\theta|\leq 1$

$|\bar{a}||\bar{b}|\cos\theta\leq|\bar{a}||\bar{b}|$

$\therefore |\bar{a}\cdot\bar{b}|\leq|\bar{a}||\bar{b}|$

10.

$$4\hat{i} - 3\hat{j} + \hat{k} = (4, -3, 1)$$

Any vector parallel to xy-plane must have z-component = 0 $\therefore$ of form $(x, y, 0)$

A vector $\perp$ to $(4, -3, 1)$ and in xy-plane must have $z = 0$

$\therefore$ We have $\bar{u} = (3, 4, 0)$

$$\therefore \hat{u} = \frac{1}{|\bar{u}|}\bar{u}$$

$$= \frac{1}{5}\bar{u}$$

$$= \left(\frac{3}{5}, \frac{4}{5}, 0\right)$$

11. $\bar{a} = (1, 3, -2)$

$\therefore |\bar{a}| = \sqrt{14}$

$\bar{b} = (k, 2, 1)$

$\therefore |\bar{b}| = \sqrt{k^2 + 5}$

$$\bar{a} \cdot \bar{b} = |\bar{a}|\,|\bar{b}| \cos 60$$

$$(1, 3, -2) \cdot (k, 2, 1) = \sqrt{14}\sqrt{k^2 + 5} \cos 60$$

$$k + 4 = \sqrt{14}\sqrt{k^2 + 5}\left(\frac{1}{2}\right)$$

$$2k + 8 = \sqrt{14}\sqrt{k^2 + 5}$$

$$4k^2 + 32k + 64 = 14k^2 + 70$$

$$0 = 10k^2 - 32k + 6$$

$$\therefore k = \frac{1}{5}, 3$$

12. $\bar{E} = 45.6N[E\,30^\circ S]$

$\therefore \bar{R} = 45.6N[W\,30^\circ N]$

$\therefore 65N[S\,35^\circ E] + 50N[E\,15^\circ N] + 110N[N\,55^\circ W] + \bar{x}$

$= 45.6N[W\,30^\circ N]$

$(37.3, -53.2) + (48.3, 12.9) + (-90.1, 63.1) + \bar{x}$

$= (-39.5, 22.8)$

$\therefore \bar{x} = (-39.5, 22.8) - (37.3, -53.2) - (48.3, 12.9) - (-90.1, 63.1)$

$= (-35, 0)$

$= 35N[W]$