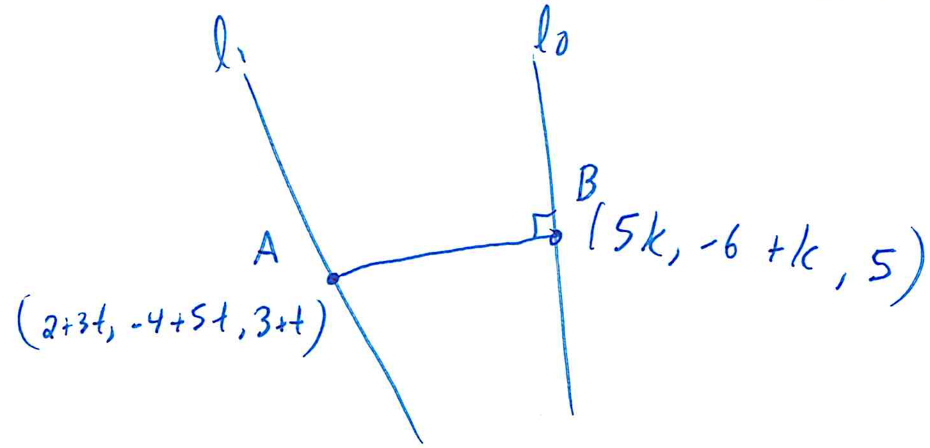
Challenge Set #3

1) If π is parallel to l_1, l_2, $\vec{n}$ is $\perp$ to $\vec{d_1}, \vec{d_2}$

$\therefore \bar{n} = \bar{d_1} \times \bar{d_2}$

$= (1, -5, 22)$

So we have $x - 5y + 22z + D = 0$. We need a pt that is the midpoint of the line segment that represents the shortest distance between l_1 + l_2.



$\bar{AB} = (5k - 3t - 2, k - 5t - 2, -t + 2)$

$\bar{AB} \cdot \bar{d_1} = 0$

$\therefore \quad 15k - 9t - 6 + 5k - 25t - 10 - t + 2 = 0$

$20k - 35t = 14$

$\bar{AB} \cdot \bar{d_2} = 0$

$25k - 15t - 10 + k - 5t - 2 = 0$

$26k - 20t = 12$

$k = \frac{14}{51}$

$t = \frac{-62}{255}$

$$A = \left(\frac{324}{255}, \frac{-266}{51}, \frac{703}{255} \right)$$

$$B = \left(\frac{70}{51}, \frac{-292}{51}, 5 \right)$$

Let M be midpt of AB

$$\therefore M = \left(\frac{17187}{13005}, \frac{-279}{51}, \frac{989}{255} \right)$$

sub M into π

$$x - 5y + 22z + D = 0$$

$$x - 5y + 22z - 114 = 0$$

2.

π has a direction vector $(2,1,2)$ from l_1

π will also have a direction vector that is $\perp$ both l_1, l_2

$$\therefore \bar{d} = (2,1,2) \times (4,2,-1)$$
$$= (-5, 10, 0)$$

lets use $(1,-2,0)$

$\therefore$ π has a normal vector

$$\bar{n} = (1,-2,0) \times (2,1,2)$$
$$= (4, 2, -5)$$

π must also contain $(3,4,1)$, a pt from l_1

$$\therefore \pi: \; 4x + 2y - 5z + D = 0$$
$$4x + 2y - 5z - 15 = 0$$

3. If π contains $l_1 + l_2$ then l_1 must intersect l_2.

You want to test this 1st.

Option A: Find POI, if it exists,

Option B: Find distance between $l_1 + l_2$ and verify it is zero.

The lines are skew, a plane does not exist