Activity 9 – Curve Sketching   Date: ____________________

DEFINITIONS: ① Local maximum: a function value (y value) that is greater than or equal to all other function values in its neighbourhood. Usually, the function changes from increasing to decreasing at this point, thus the derivative changes from positive to negative at this point.
Formal notation: $f(x)$ has a local maximum at $x = c$, if $f(c) \geq f(x)$ for all x close to c.
The local maximum value is $f(c)$.

② Local minimum: a function value (y value) that is less than or equal to all other function values in its neighbourhood. Usually, the function changes from decreasing to increasing at this point, thus the derivative changes from positive to negative at this point.
Formal notation: $f(x)$ has a local minimum at $x = c$, if $f(c) \leq f(x)$ for all x close to c.
The local minimum value is $f(c)$.

③ Absolute maximum: a function value (y value) that is greater than or equal to all other function values.
Formal notation: $f(x)$ has an absolute maximum at $x = d$, if $f(d) \geq f(x)$ for all x in D_f.
The absolute maximum value is $f(d)$.

④ Absolute minimum: a function value (y value) that is less than or equal to all other function values.
Formal notation: $f(x)$ has an absolute minimum at $x = d$, if $f(d) \leq f(x)$ for all x in D_f.
The absolute minimum value is $f(d)$.

⑤ A function is increasing (over an interval) if the graph of a function RISES from left to right along the x-axis (over said interval).
Formal notation: $f(x_1) < f(x_2)$, when $x_1 < x_2$

⑥ A function is decreasing (over an interval) if the graph of a function FALLS from left to right along the x-axis (over said interval).
Formal notation: $f(x_1) > f(x_2)$, when $x_1 < x_2$

Properties of the first derivative of functions:

1. $f'(x) = 0$ gives x value of possible local extrema
2. $f'(x) > 0$ on an interval, then $f(x)$ is increasing on that interval
3. $f'(x) < 0$ on an interval, then $f(x)$ is decreasing on that interval

The first derivative test:

1. If $f'(a) = 0$ and $f(x)$ goes from increasing to decreasing then $x = a$ is a local maximum.
2. If $f'(a) = 0$ and $f(x)$ goes from decreasing to increasing then $x = a$ is a local minimum.

Note: ☞A maximum or a minimum is often referred to as an extrema.

☞Not all functions will have a maximum or a minimum.

☞The function could have a point where the derivative = 0 but the derivative does not change signs, then the point is neither a maximum nor a minimum.

Properties of the second derivative of functions:

1. $f''(x)=0$ may give points of inflection (places where the function changes from concave up to concave down or vice versa)
2. $f''(x)>0$ on an interval, then $f(x)$ is concave up on that interval
3. $f''(x)<0$ on an interval, then $f(x)$ is concave down on that interval

The second derivative test:

For a function $f(x)$, where $f''(x)$ exists on an interval containing *a*

1. If $f'(a)$=0 and $f''(a)>0$ then $x=a$ is a local minimum.
2. If $f'(a)$=0 and $f''(a)<0$ then $x=a$ is a local maximum.

Curve Sketching Given Information

1. Use the information to sketch the graph.

a) The domain of the function is the set of all real numbers except $x=1$ and $x=-1$.

b) $\lim\limits_{x\to 1^+} f(x)=-\infty$ $\lim\limits_{x\to 1^-} f(x)=\infty$ $\lim\limits_{x\to -1^+} f(x)=\infty$ $\lim\limits_{x\to -1^-} f(x)=-\infty$

c) $\lim\limits_{x\to\infty} f(x)=0$, $f(x)<0$ as $x\to\infty$

d) $\lim\limits_{x\to-\infty} f(x)=0$, $f(x)<0$ as $x\to-\infty$

e) The y intercept is 2. There are no x intercepts.

2. Sketch a graph of a polynomial (continuous) function that satisfies all of the following conditions:

a) $f'(x)>0$ for $0<x<1$ and $f'(x)<0$ for $1<x<\infty$

b) $f''(x)>0$ for $2<x<\infty$ and $f''(x)<0$ for $0<x<2$

c) $\lim\limits_{x\to\infty} f(x)=0$

d) $f(x)$ is an odd function (symmetrical about the origin)

3. Sketch a graph of a (continuous) function that satisfies all of the following conditions:

a) $f(0)=f(3)=2,\quad f(-1)=f(1)=0$

b) $f'(-1)=f'(1)=0$

c) $f'(x)<0$ for $-\infty<x<-1$ and for $0<x<1$

d) $f'(x)>0$ for $-1<x<0$ and for $1<x<\infty$

e) $f''(x)>0$ for $x<3\ (x\neq 0)$ and $f''(x)<0$ for $3<x<\infty$

f) $\lim\limits_{x\to\infty} f(x)=4$ and $\lim\limits_{x\to-\infty} f(x)=\infty$

Curve sketching Given Equation

A complete analysis contains: intercepts, asymptotes, domain, Max/Min points, intervals of Inc/Dec, inflection points, intervals of CU/CD, and an accurate graph.

Complete a full analysis on the following functions and graph:

1. $f(x)=36x-3x^2-2x^3$	2. $y=\frac{4x}{x^2+9}$	3. $k(x)=4x^2-2x^4-3$
4. $g(x)=\frac{4-x^2}{4+x^2}$	5. $g(x)=x^4-2x^3+1$	6. $f(x)=\frac{x^2}{x-3}$
7. $y=2x^2e^x$	8. $k(x)=\frac{x^2}{e^{2x}}$	9. $h(x)=x^5-10x^4$
10. $j(x)=\frac{\ln x}{x^3}$	11. $f(x)=6x^2-\frac{1}{4}x^4$	12. $y=\frac{x}{x^2-9}$

Solutions:

1. Max (2, 44) Min (-3,-81) IP $\left(\frac{-1}{2},\frac{-37}{2}\right)$	2. Max (3, 0.7) Min (-3, -0.7) IP (0, 0) $\left(\sqrt{27},0.6\right)$ $\left(-\sqrt{27},-0.6\right)$ HA $y=0$	3. Max (1, -1) (-1, -1) Min (0, -3) IP $(\pm 0.6,-1.9)$
4. Max (0, 1) IP $(\pm 1.2, 0.5)$ HA $y=-1$	5. Min (1.5, -0.7) IP (1, 0) (0, 1)	6. Max (0, 0) Min (6, 12) VA $x=3$, OA $y=x+3$
7. IP (-3.414, 0.767) (-0.586, 0.382) $CU\ x<-3.414\ or\ x>-0.586$ $CD\ -3.414<x<-0.586$	8. Max (1, 0.14) Min (0, 0) IP (1.7, 0.1) (0.3, 0.05) HA $y=0$	9. Max (0, 0) Min (8, -8192) IP (6, -5184)
10. Max (1.4, 0.12) IP (1.8, 0.1) VA $x=0$ HA $y=0$	11. Max $(\pm\sqrt{12},36)$ Min (0, 0) IP $(\pm 2,20)$	12. No Max Min IP (0, 0) VA $x=\pm 3$ HA $y=0$