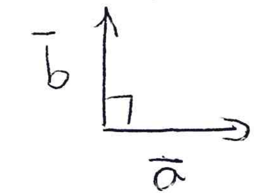
Challenge Set #3

1) Let $\bar{a}, \bar{b}$ be as shown



$$|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}| \sin\theta$$
$$= (1)(1) \sin 90$$
$$= 1$$

2). $$|\bar{a} \times \bar{b}| = \sqrt{(\bar{a} \cdot a)(\bar{b} \cdot \bar{b}) - (\bar{a} \cdot \bar{b})^2}$$

$$RS = \sqrt{|\bar{a}|^2 |\bar{b}|^2 - (|\bar{a}||\bar{b}| \cos\theta)^2}$$
$$= \sqrt{|\bar{a}|^2 |\bar{b}|^2 - |\bar{a}|^2 |\bar{b}|^2 \cos^2\theta}$$
$$= \sqrt{|\bar{a}|^2 |\bar{b}|^2 (1 - \cos^2\theta)}$$
$$= \sqrt{|\bar{a}|^2 |\bar{b}|^2 \sin^2\theta}$$
$$= |\bar{a}||\bar{b}| \sin\theta$$
$$= |\bar{a} \times \bar{b}|$$
$$= LS$$

3. Let $\bar{u} = (1,0,1)$

$\bar{v} = (0,1,1)$

$\bar{w} = (1,1,0)$

$$LS = \bar{u} \times (\bar{v} \times \bar{w})$$
$$= (1,0,1) \times (-1, 1, -1)$$
$$= (-1, 0, 1)$$

$$RS = (\bar{u} \times \bar{v}) \times \bar{w}$$
$$= (-1, -1, 1) \times (1, 1, 0)$$
$$= (-1, 1, 0)$$

$$\therefore LS \neq RS$$

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$LS = \bar{u} \cdot (\bar{v} \times \bar{w})$
$= (1,0,1) \cdot (-1,1,-1)$
$= -2$

$RS = (\bar{u} \times \bar{v}) \circ \bar{w}$
$= (-1,-1,1) \cdot (1,1,0)$
$= -2$

$LS = RS$

4) Let $\bar{u} = (u_1, u_2, u_3$, $\bar{v} = (v_1, v_2, v_3)$, $\bar{w} = (w_1, w_2, w_3)$

$LS = \bar{u} \cdot (\bar{v} \times \bar{w})$

$= (u_1, u_2, u_3) \circ (v_2 w_3 - w_2 v_3 , w_1 v_3 - v_1 w_3 , v_1 w_2 - w_1 v_2)$

$= u_1 v_2 w_3 - u_1 w_2 v_3 + u_2 w_1 v_3 - u_2 v_1 w_3 + u_3 v_1 w_2 - u_3 w_1 v_2$

$= u_2 v_3 w_1 - u_3 v_2 w_1 + u_3 v_1 w_2 - u_1 w_2 v_3 + u_1 v_2 w_3 - u_2 v_1 w_3$

$= (u_2 v_3 - u_3 v_2 , u_3 v_1 - u_1 v_3 , u_1 v_2 - u_2 v_1) \cdot (w_1, w_2, w_3)$

$= (\bar{u} \times \bar{v}) \circ \bar{w}$

5) $(\bar{a}-\bar{b})\times(\bar{a}+\bar{b}) = 2\bar{a}\times\bar{b}$

$LS = \bar{a}\times\bar{a} + \bar{a}\times\bar{b} - \bar{b}\times\bar{a} - \bar{b}\times\bar{b}$

$= \bar{a}\times\bar{b} + \bar{a}\times\bar{b}$

$= 2\bar{a}\times\bar{b}$

$= RS$

6. $\bar{u} = (3,-2,1)$

$\therefore \bar{x} = (0,1,2)$

$\bar{y} = (1,0-3)$

$\bar{z} = (2,3,0)$

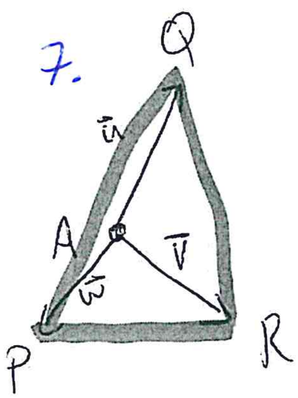
$(\bar{x}\times\bar{y})\cdot\bar{z} = (-3,2,-1)\cdot(2,3,0)$

$= -6+6+0$

$= 0$

$\therefore \bar{x},\bar{y},\bar{z}$ are coplanar

7.



From pt P, the triangle has vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$

$\overrightarrow{PQ} = -\bar{w}+\bar{u}$

$\overrightarrow{PR} = -\bar{w}+\bar{v}$

$A = \frac{1}{2}\left|(-\bar{w}+\bar{u})\times(-\bar{w}+\bar{v})\right|$

$= \frac{1}{2}\left|\bar{w}\times\bar{w} - \bar{w}\times\bar{v} - \bar{u}\times\bar{w} + \bar{u}\times\bar{v}\right|$

$= \frac{1}{2}\left(\bar{v}\times\bar{w} + \bar{w}\times\bar{u} + \bar{u}\times\bar{v}\right)$

8. If $(\bar{a} \times \bar{b}) \times \bar{c}$ lies in same plane of $\bar{a}$ and $\bar{b}$

then I want to show $[(\bar{a} \times \bar{b}) \times \bar{c}] \cdot (\bar{a} \times \bar{b}) = 0$

If I let $\bar{d} = \bar{a} \times \bar{b}$

this becomes

$$(\bar{d} \times \bar{c}) \cdot \bar{d} = 0$$

which is clearly true

because $\bar{d} \times \bar{c}$ is $\perp$ to $\bar{d}$ & $\bar{c}$

9 $\because$ $\bar{x}, \bar{y}, \bar{z}$ are all $\perp$ to each other

$$\bar{x} \cdot \bar{y} = \bar{x} \cdot \bar{z} = \bar{y} \cdot \bar{z} = 0$$

① $\bar{x} \cdot \bar{y} = a + 2b - 6 = 0$

$a + 2b = 6$

② $\bar{x} \cdot \bar{z} = -7a + 22 + 3c = 0$

$-7a + 3c = -22$

③ $\bar{y} \cdot \bar{z} = -7 + 11b - 2c = 0$

$11b - 2c = 7$

For ① + ②

$$\begin{array}{r} -7(a + 2b = 6) \\ - \quad -7a + 3c = -22 \\ \hline ④ \quad -14b - 3c = -20 \end{array}$$

for ④ + ③

$$\begin{array}{r} 2(-14b - 3c = -20) \\ - \quad 3(11b - 2c = 7) \\ \hline -61b = -61 \end{array}$$

$\therefore b = 1, c = 2, a = 4$

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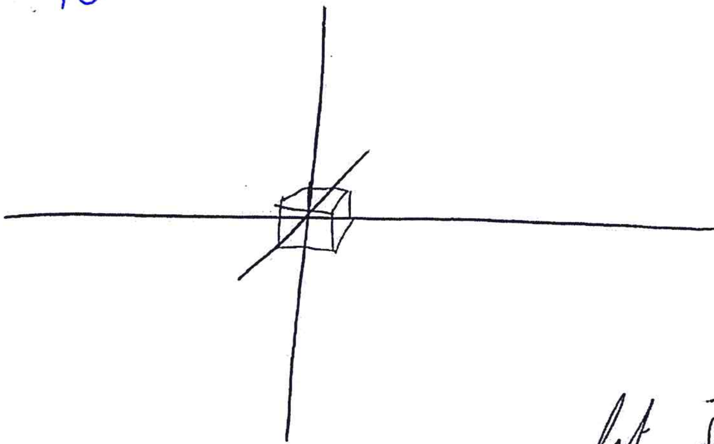

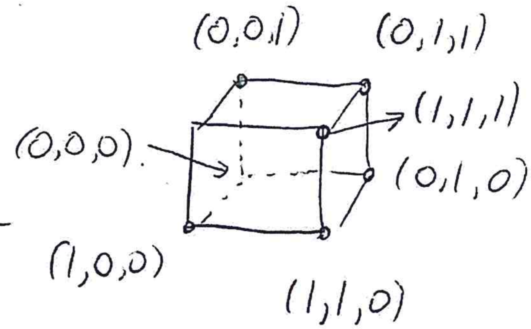

let $\bar{f}$ = face diagonal = $(1,1,0) - (0,0,0)$

$= (1,1,0)$

let $\bar{b}$ = body diagonal = $(1,1,1)$

$$\therefore \cos\theta = \frac{\bar{f}\cdot\bar{b}}{|\vec{f}|\,|\bar{b}|}$$

$$= \frac{2}{\sqrt{2}\sqrt{3}}$$

$$\theta = 35.3^\circ$$

11.

$$\bar{F} = 15\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right) \leftarrow \hat{v}$$

$$= (5, 10, 10)$$

$$\bar{d}_1 = \bar{OP} = (1, -3, 4)$$

$$\bar{d}_2 = \bar{PA} = (6, 5, 1)$$

$$\therefore W = \bar{F} \cdot \bar{d} + \bar{F} \cdot \bar{d}_2$$

$$= \bar{F} \cdot (\bar{d}_1 + \bar{d}_2) \longleftarrow \quad \bar{d}_R = \text{resultant} = (7, 2, 5)$$

$$= \bar{F} \cdot \bar{d}_R$$

$$= (5, 10, 10) \cdot (7, 2, 5)$$

$$= 105 \text{ J}$$

12. $$W = \bar{F} \cdot \bar{d}_1 + \bar{F} \cdot \bar{d}_2 + \bar{F} \cdot d_3$$

$$= \bar{F} \cdot \bar{d}_R$$

$$= (5, 10, 10) \cdot (7, 2, 5)$$

$$= 105 \text{ J}$$

(same as # 11)