Challenge Set #3

1) If $\vec{a}$ and $\vec{b}$ are perpendicular unit vectors, what is the length of $\vec{a}\times\vec{b}$?

2) Prove that $\left|\vec{a}\times\vec{b}\right| = \sqrt{(\vec{a}\cdot\vec{a})(\vec{b}\cdot\vec{b}) - (\vec{a}\cdot\vec{b})^2}$.

3) Show that $\vec{u}\times(\vec{v}\times\vec{w}) \neq (\vec{u}\times\vec{v})\times\vec{w}$.

4) Show that $\vec{u}\cdot(\vec{v}\times\vec{w}) = (\vec{u}\times\vec{v})\cdot\vec{w}$.

5) Prove that $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b}) = 2\vec{a}\times\vec{b}$ for all $\vec{a},\vec{b}\in\Re^3$.

6) The vector $\vec{u} = (3,-2,1)$ is given. Determine any three non-collinear vectors that are perpendicular to $\vec{u}$. Show that the three vectors you determined are coplanar.

7) Three vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ have a common initial point. Their endpoints form a triangle. Prove that the magnitude of the vector $\frac{1}{2}(\vec{u}\times\vec{v}+\vec{v}\times\vec{w}+\vec{w}\times\vec{u})$ is equal to the area of the triangle.

8) For any vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, explain why $(\vec{a}\times\vec{b})\times\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$.

9) Given the vectors $\vec{x} = (a,2,3)$, $\vec{y} = (1,b,-2)$ and $\vec{z} = (-7,11,c)$ are mutually perpendicular vectors, find a, b and c.

10) A body diagonal of a unit cube is a line through the centre joining opposite vertices. A face diagonal is a diagonal along a face of the cube. Find the angle between a body diagonal and a face diagonal of a unit cube.

11) Find the total work done by a 15 N force, $\vec{F}$, in the direction of vector (1, 2, 2) when it moves a particle from O(0, 0, 0) to P(1, -3, 4) and then from P to A(7, 2, 5). The distance is measured in metres.

12) Find the total work done by a 15 N force, $\vec{F}$, in the direction of vector (1, 2, 2) when it moves a particle from O(0, 0, 0) to R(-2, 4, 0) and then from R to S(11, 1, -1), and then from S to A(7, 2, 5). The distance is measured in metres.