MCV 4U Planes Review

1. A plane is defined by the points A(1,-2,4), B(3,5,-3) and C(-2,-7,1)
   a) Give two direction vectors for this plane
   b) Find a normal for this plane
   c) state the vector equation for this plane
   d) state the parametric equations for this plane
   e) state the scalar equation for this plane

Answers: a) $\vec{d} = (2,7,-7)$ $\quad \vec{e} = (-3,-5,-3)$
b) $\vec{n} = (-56,27,11)$
c) $(x, y, z) = (1,-2,4) + t(2,7,-7) + s(-3,-5,-3)$
d) $x = 1 + 2t - 3s$, $y = -2 + 7t - 5s$, $z = 4 - 7t - 3s$
e) $56x - 27y - 11z - 66 = 0$

2. Give parametric equations for the plane $\pi : 2x - 3y + z - 6 = 0$. Ans: $x = 3s, y = t + 2s, z = 6 + 3t$

3. Find the angles between the planes $\begin{matrix} \pi_1 : 3x - 2y + 5z = 12 \\ \pi_2 : x - 5y + z = 3 \end{matrix}$ Ans: acute is 55.8^o and obtuse is 124.2^o

4. Given the planes: $\begin{matrix} \pi_1 : (2k-4)x - ky + z = 15 \\ \pi_2 : kx + (k-2)y - kz = 10 \end{matrix}$

   Find value(s) of k that would make the planes perpendicular. $k = 0$ or $k = 3$

5. Explain why the point $P(2,21,8)$ and the line $\vec{r} = (-4,-3,-1) + t(2,8,3)$ do not determine a plane.
Ans: show that P lines on the line therefore P and line are collinear therefore do not define the plane

6. Find the intersection of the following lines. Then classify them as an inconsistent or consistent, dependent or independent system.

   a) $\begin{matrix} \ell_1 : (x, y) = (-1,3) + t(-2,4) \\ \ell_2 : 2x + y - 1 = 0 \end{matrix}$

   $(t, 1-2t), \quad t \in \Re$

   b) $\ell_1 : (x, y, z) = (-1,3,5) + t(1,-2,6)$ and $\ell_2 : \begin{cases} x = -13 + 3k \\ y = -8 + k \\ z = -1 + 3k \end{cases}$

   { system is inconsistent }

   c) $\ell_1 : \dfrac{x-2}{1} = \dfrac{y-1}{-1} = \dfrac{z}{1}$
   $\ell_2 : \dfrac{x-3}{2} = \dfrac{y}{3} = \dfrac{1-z}{1}$

   { (3, 0, 1)}

   d) $\ell_1 : \begin{cases} x = 1 + t \\ y = 1 + 2t \\ z = 1 - 3t \end{cases}$ and $\ell_2 : \begin{cases} x = 3 - 2u \\ y = 5 - 4u \\ z = -5 + 6u \end{cases}$

   $(a, 2a-1, 4-3a) \; a \in \Re$

7. Show that the lines $\vec{r} = (4, 7, -1) + t(4, 8, -4)$ and $\vec{r} = (1, 5, 4) + u(-1, 2, 3)$ intersect at right angles and find the point of intersection. { (2, 3, 1)}

8. Find the distance between:
   a) the point $A(-6,5,-3)$ and the line $(x, y, z) = (6,1,3) + t(5,-3,3)$ { 2.76 units}
   b) the lines $\ell_1 : \vec{r} = (1,6,-2) + t(1,-2,5)$ $\quad \ell_2 : \vec{r} = (3,-4,-9) + k(-2,7,1)$ { 0.39 units}
   c) the parallel lines: $\ell_1 : \dfrac{x-1}{2} = \dfrac{y+4}{1}, z = 1$ and $\ell_2 : \begin{cases} x = 4t \\ y = 1 + 2t \\ z = 6 \end{cases}$ { 7.01 units}

9. Solve each system. For each set state the geometrical interpretation between the planes (what case is it?).

a) $\pi_1: \; 2x - 2y + 4z = 5$
$\pi_2: \; x - y + 2z = 2$
{ system is inconsistent}

b) $\pi_1: \; 3x + 2y - 4z + 1 = 0$
$\pi_2: \; 2x - y - z + 3 = 0$
$\{ \; (x, y, z) = (-1,1,0) + t\left(\frac{6}{7}, \frac{-5}{7}, 1\right), \quad t \in \Re \; \}$

c) $\pi_1: 3x - y - 2z = 3$
$\pi_2: 6x - 2y - 4z = 4$
$\pi_3: 9x - 3y - 6z = 5$
{ system is inconsistent}

d) $\pi_1: x + 2y - z = 3$
$\pi_2: 3x + y + z = -2$
$\pi_3: 2x - y + 2z = 4$
{ system is inconsistent }

e) $\pi_1: x + 2y - z = 3$
$\pi_2: 3x + y + z = -2$
$\pi_3: 2x - y + 2z = -5$
$\{ \; (x, y, z) = \left(\frac{-7}{5}, \frac{11}{5}, 0\right) + t\left(\frac{-3}{5}, \frac{4}{5}, 1\right), \quad t \in \Re \; \}$

f) $\pi_1: x + 5y + z = -10$
$\pi_2: 3x + 2y + z = 2$
$\pi_3: 25x - 15y - 5z = -2$
$\{ \; \left(\frac{-18}{55}, \frac{-232}{55}, \frac{628}{55}\right) \}$

10. Find the distance between

a) the (parallel) planes $\pi_1: \; x + 2y - 5z - 10 = 0$ and $\pi_2: \; 2x + 4y - 10z - 17 = 0$ { 0.27 units }

b) the line $\vec{r} = (1, 3, 2) + t(1, 2, -1)$ and the plane $y + 2z = 5$ { 0.89 units }

c) from the point A(-1,3,4) to the plane $2x - y - 5z - 10 = 0$. Does the point lie above or below the plane? Justify your reasoning. { 6.39 units, above}

11. a) Find the intersection of the line $\ell: (x, y, z) = (3,1,-1) + t(-2,-3,5)$ and the plane $2x + y - 3z = -34$
{ (-1, -5, 9) }

b) Find the parametric equations of the line of intersection of the planes
$\pi_1: 5x - 2y + z - 12 = 0$
$\pi_2: 3x - y + 2z - 8 = 0$
Show that the direction vector for the line of intersection is a scalar multiple of the cross product of the normals for the two planes. { $x = 4 - 3t, \; y = 4 - 7t, \; z = t, \; t \in \Re$ }

12. In the following system of equations, k is a real number.

$\pi_1: -2x + y + z = k + 1$ $\pi_2: kx + z = 0$ $\pi_3: y + kz = 0$	For what value(s) of k does the system i) have no solution? { k = 2 } ii) have exactly one solution? { $k \neq 2, -1$ } iii) have an infinite number of solutions? {k = - 1}

13. For each set of three planes determine the intersection (if any). For each set classify the system as consistent or inconsistent, and (if possible) dependent or independent. For each set state the geometrical interpretation between the planes (what case is it?).

$$SET\ 1: \begin{cases} \pi_1: 2x - y - z + 4 = 0 \\ \pi_2: 3x - 2y + 3z - 5 = 0 \\ \pi_3: 13x - 8y + 7z - 7 = 0 \end{cases}$$

$\{ \; (x, y, z) = (-13, -22, 0) + t(5, 9, 1), \quad t \in \Re \}$

$$SET\ 2: \begin{cases} \pi_1: 3x - 2y + z - 12 = 0 \\ \pi_2: x + y - 2z + 7 = 0 \\ \pi_3: 2x + y - 3z + 9 = 0 \end{cases}$$

{ (2, -1, 4) }