Challenge Set #3

1) Answers vary

   1) Select 2, non-collinear direction vectors
   2) Select any pt for l_1
   3) Select any pt not on l_1 for l_2

2) l_1: $x = k$, $y = k$, $z = -2 + k$ l_2: $x = -2m$, $y = -3 + m$, $z = 3m$

Let l_3 be the line. l_3 must pass through $(0,1,2)$, l_1 and l_2

Let the direction vector for l_3 be $\bar{d} = (0,1,2) - (k, k, -2+k)$

$\therefore l_3: (x,y,z) = (0,1,2) + n(-k, 1-k, 4-k)$

l_3 must also intersect l_2

$\therefore l_3$: $x = -nk$, $y = 1 + n - nk$, $z = 2 + 4n - nk$ l_2: $x = -2m$, $y = -3 + m$, $z = 3m$

$\therefore$

$-nk = -2m$ ①

$1 + n - nk = -3 + m$ ②

$2 + 4n - nk = 3m$ ③

sub ① into ②

$$1 + n - 2m = -3 + m$$

$$n - 3m = -4$$

sub ① into ③

$$2 + 4n - 2m = 3m$$

$$4n \cancel{+} -5m = -2$$

$$\begin{array}{rl} & 4(n - 3m = -4) \\ - & 4n - 5m = -2 \\ \hline & -7m = -14 \end{array}$$

$$m = 2$$

$$\therefore n = 2$$

$$k = 2$$

$$\therefore l_3 : (x, y, z) = (0, 1, 2) + n(-2, -1, 2)$$

$$\boxed{\frac{x}{-2} = \frac{y-1}{-1} = \frac{z-2}{2}}$$

3) let l_1: $x = k + 3t$ $\quad$ l_2: $x = 1 + 3m$

$y = -4 + 2t$ $\quad$ $y = 1 - m$

$z = -6 + t$ $\quad$ $z = 2 - m$

$\therefore$ $k + 3t = 1 + 3m$

① $\boxed{3t - 3m = 1 - k}$

$-4 + 2t = 1 - m$

② $\boxed{2t + m = 5}$

$-6 + t = 2 - m$

③ $\boxed{t + m = 8}$

Solve for t, m in ②, ③

$$\begin{array}{r} 2t + m = 5 \\ - \quad t + m = 8 \\ \hline t \quad = -3 \end{array}$$

$m = 11$

If they intersect, $t = -3$, $m = 11$

a) ① $3t - 3m = 1 - k$

$3(-3) - 3(11) = 1 - k$

$k = 43$

b) $k \neq 43$

4) Let l_3 be the line. If l_3 is $\perp$ to l_1, l_2 then $\bar{d}_3 = \bar{d}_1 \times \bar{d}_2$

$$\bar{d}_3 = (1, -2, 1)$$

If l_1 and l_2 intersected, this would be a much simpler problem. The intersection would be the reference pt of l_3

It can be shown that l_1, l_2 are skew.

This is similar to #2 except we already know $\bar{d}_3$

$\therefore$ l_3 has $\bar{d} = (1, -2, 1)$ and a pt on l_1, lets say:

$$(1+3k, -1+2k, 1+k)$$

$\therefore$ l_3: $x = 1+3k+m$
$y = -1+2k-2m$
$z = 1+k+m$

l_2: $x = 2+t$
$y = -3+2t$
$z = 3t$

$\therefore$ $1+3k+m = 2+t$

① $3k+m-t = 1$

$-1+2k-2m = -3+2t$

$2k-2m-2t = -2$

② $k-m-t = -1$

$1+k+m = 3t$

③ $k+m-3t = -1$

$$\text{①} \quad 3k + m - t = 1$$
$$\text{②} \quad k - m - t = -1$$
$$-\quad\overline{}$$
$$2k + 2m = 2$$
$$k + m = 1$$

$$\text{①}\, 3(3k + m - t = 1)$$
$$-\text{③} \quad k + m - 3t = -1$$
$$\overline{}$$
$$8k + 2m = 4$$
$$4k + m = 2$$

$$k + m = 1$$
$$-\quad 4k + m = 2$$
$$\overline{}$$
$$-3k = -1$$
$$k = 1/3$$
$$\therefore m = 2/3$$
$$\therefore \quad t = 2/3$$

$$\therefore \quad l_3 : (x,y,z) = (1+3k, -1+2k, 1+k) + m(1,-2,1)$$
$$(x,y,z) = (2, -1/3, 4/3) = m(1,-2,1)$$