Activity 7 Differentiation rules, day 2 Date:_____________

Part1) <u>Derivative of a composite function</u>

The chain rule – For composite functions $y = (f \circ g)(x) = f(g(x))$
Here is how to find the derivative of a composite function:

The Chain Rule

$y = f(g(x))$ and $u = g(x)$

then

$$\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$$

or

$$\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx}$$

A proof of the chain rule can be found on pages 99 and 100.

1. Find the derivative of each of the following using the chain rule.

a) $f(x) = (2x+3)^4$
b) $f(x) = (4x+1)^{\frac{1}{2}}$
c) $y = (x^2 + x)^{-1}$
d) $g(x) = e^{3x}$
e) $f(t) = e^{3t-5}$
f) $h(x) = e^{5-6x+x^2}$
g) $y = \ln(5x)$
h) $\frac{d}{dx}(\ln(x^2 + e^x))$
i) $y = \ln\sqrt{x+1}$
j) $y = e^{\ln x}$
k) $f(x) = (\ln x + 2x)^{\frac{1}{3}}$
l) $h(x) = \frac{1}{(3-4x)^2}$
m) $y = \sin(2x)$
n) $g(x) = \frac{1}{\cos 3x}$
o) $k(x) = \sin\sqrt{x}$
p) $f(x) = 7\sin(3x+5)$
q) $y = \frac{1}{3}\cos^3 4x$
r) $y = 5e^{-4x}$
s) $f(x) = \sqrt{1+e^x}$
t) $y = x + e^{\sqrt{x}}$
u) $f(x) = 3^{3x}$
v) $h(x) = 0.2^{-x} + x^{-2}$
w) $k(x) = 10^{-x+2}$
x) $f(x) = (x^2 + x)^{\frac{3}{2}}$
y) $y = \ln(3x^2 - 4x + 3)^4$
z) $f(x) = \sin^4(x^3 - 3x)^2$
aa) $y = \ln(\sin 3x)$
bb) $g(x) = 2^{\sqrt{x^2+1}}$

2. For what value(s) of x do the curves $y = (1+x^3)^2$ and $y = 2x^6$ have tangents that are parallel?

3. Find the slope of the tangent to the curve $y = (3x + x^2)^{-2}$ at the point $B\left(-2, \frac{1}{4}\right)$.

4. Find the equation of the tangent to the curve $y = (x^3 - 7)^5$ at $x = 2$.

5. Find the slope of the tangent line to the graph of $y = (9 - x^2)^{\frac{2}{3}}$ at the point $K(1,4)$.

6. For $f(x) = \frac{1}{4\sqrt{1+x}}$, find the point of contact of all tangents with slope equal to - 1.

7. Find the equation of the tangent line to $y = \cos^2 x$ at $x = \frac{\pi}{3}$.

8. Find the slope of the tangent line to $f(x) = \sqrt{\cos x}$ at $x = \frac{\pi}{3}$.

9. Find the slope of the tangent to $g(x) = \sin^2 x + \sin x + 1$ at the point where $x = \frac{\pi}{6}$.

10. Find the equation of the tangent line to $y = 5x - 3x^2 - \ln(3x-5)$ at the point where $x = 2$.

Part 2) Second derivative:

A) Why would we need to look at the derivative of a derivative? What does the second derivative tell you about a function?

Notation for the second derivative: $f''(x) = y'' = \dfrac{d^2 f(x)}{dx^2} = \dfrac{d^2 y}{dx^2}$.

1. Find the second derivative for each of the following:

a) $f(x) = 4x^2 - 5x + 7$	b) $y = 4t^5 - 9t^3 + 2$	c) $f(x) = 5(2x^2 - 3)^4$	d) $y = \dfrac{3x-5}{1-2x}$
e) $g(x) = \dfrac{x}{x^3 - 2}$	f) $y = \dfrac{x}{\sqrt{x^2+1}}$	g) $h(x) = \dfrac{5x}{x^2+1}$	h) $y = \dfrac{4(x-1)}{x^2}$

2. Find the second degree polynomial $f(x)$ given that $f(2) = 1$, $f'(2) = 14$ and $f''(2) = 10$.

3. Find the value of the first and second derivatives to the function $y = \sqrt{x^2+7} + x^2$ at the point where $x = 3$.

4. Find the value of the first and second derivatives to the function $f(x) = 3x\sqrt{2x+5}$ at the point where $x = 2$.

5. Show that $y = 2x^3 + 3x^2$ satisfies the equation $x^2 \dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = 0$.

Answers:

1. a) $f''(x) = 8$	b) $y'' = 80t^3 - 54t$
c) $f''(x) = 80(2x^2-3)^2(14x^2-3)$	d) $\dfrac{d^2y}{dx^2} = \dfrac{-28}{(1-2x)^3}$
e) $\dfrac{d^2g(x)}{dx^2} = \dfrac{6x^5 + 24x^2}{(x^3-2)^3}$	f) $\dfrac{d^2y}{dx^2} = \dfrac{-3x}{\sqrt{(x^2+1)^5}}$
g) $h'(x) = \dfrac{10x(x^2-3)}{(x^2+1)^3}$	h) $y'' = \dfrac{8(x-3)}{x^4}$

2. $f(x) = 5x^2 - 6x - 7$

3. $y'(3) = \dfrac{27}{4}$, $y''(3) = \dfrac{135}{64}$

4. $f'(2) = 11$, $f''(2) = \dfrac{16}{9}$