Challenge set #1

1. If $\hat{a}$ and $\hat{b}$ are unit vectors that make an angle of $60°$ with each other, calculate
   a) $\left|3\hat{a}-5\hat{b}\right|$ b) $\left|8\hat{a}+3\hat{b}\right|$

2. What conditions must be satisfied by the non-zero vectors $\vec{u}$ and $\vec{v}$ for the following to be true?

a) $|\vec{u}+\vec{v}|=|\vec{u}-\vec{v}|$ b) $|\vec{u}+\vec{v}|=|\vec{u}|+|\vec{v}|$ c) $|\vec{u}+\vec{v}|=|\vec{u}|-|\vec{v}|$ d) $|\vec{u}-\vec{v}|=|\vec{u}|+|\vec{v}|$

3. ABCDEF is a regular hexagon with sides of unit length. Find the magnitude and direction of $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$

4. If $|\vec{x}|=11, |\vec{y}|=23$ and $|\vec{x}-\vec{y}|=30$, find $|\vec{x}+\vec{y}|$

5. If three vertices of a parallelogram are (-5, 3), (5, 2) and (7, -8), determine all possible coordinates of the fourth vertex.

6. Show that the three points A(1, 1, -2), B(10, 1, -8) and C(-2, 1, 0) are collinear.

7. A triangle has vertices A(-1, 3, 4), B(3, -1, 1) and C(5, 1, 1). Show that this is a right triangle.

8. The resultant of two vectors $\vec{a}$ and $\vec{b}$ is the vector $\vec{r}=5[E53°S]$. If $\vec{a}=\sqrt{29}[E21.8°N]$, determine $\vec{b}$.

9. The diagonals of parallelogram ABCD meet at the point E. Show that $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}=\vec{0}$

10. Recall that the three medians of a triangle ABC intersect in a point G and that G divides each median in a 2:1 ratio. Show that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0}$.