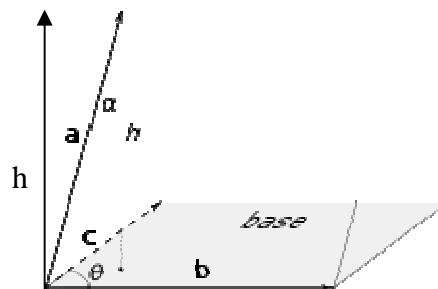Applications of Cross Product and Dot Product Part 2

 # 5 The Volume of a Parallelepiped

$$V_{ppp'd} = |\vec{u} \times \vec{v} \cdot \vec{w}|$$
$$V = A_{base}h$$
$$= (A_{parallelopiped})(height)$$
$$= |\vec{b} \times \vec{c}| \, h$$
$$= |\vec{b} \times \vec{c}| \, |\vec{a}| \cos \alpha$$
$$= |(\vec{b} \times \vec{c}) \bullet \vec{a}|$$

Therefore, the volume of a parallelepiped is equal to the absolute value of the triple scalar product of the three vectors which make up the parallelepiped.

Ex. Find the volume of the parallelepiped with the vectors $\vec{a} = (-1,4,4), \vec{b} = (3,-3,2) \, and \, \vec{c} = (4,-2,7)$ as the vectors that generate the parallelepiped.
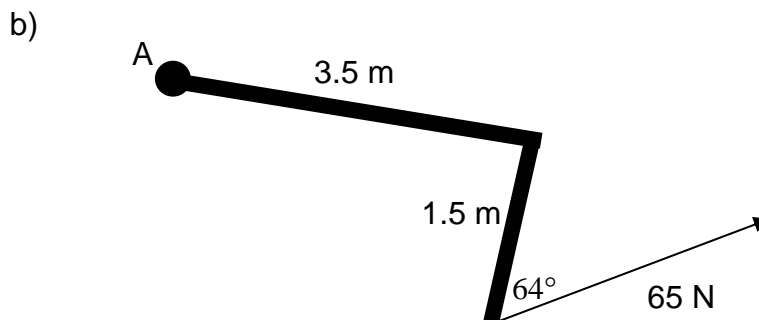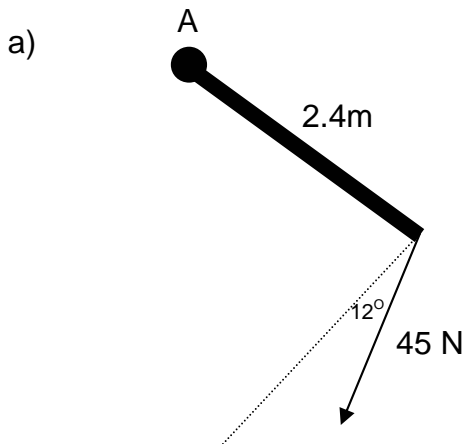
#6 Moment

$$\vec{M} = \vec{r} \times \vec{F} \qquad\qquad |\vec{M}| = |\vec{r} \times \vec{F}|$$
$$= |\vec{r}||\vec{F}| \sin \theta \, \hat{n} \qquad\qquad = |\vec{r}||\vec{F}| \sin \theta$$

Sometimes torque is referred to as the Moment and the magnitude of the torque is referred to as the magnitude of the moment.

Ex. Find the moment and the magnitude of the moment about the point A in each of the following diagrams:

a)

A

2.4m

12°

45 N

b)

A

3.5 m

1.5 m

64°

65 N

7 Torque

The term of torque is usually used when two equal and opposite forces that rotate an object. However, the terms torque and moment are usually interchangeable.

$$\vec{T} = \vec{r} \times \vec{F}$$

$$= |\vec{r}||\vec{F}| \sin\theta\, \hat{n}$$

where $\vec{F}$ is the force applied, $\vec{r}$ is the lever arm from the point of rotation, θ is the angle between $\vec{F}$ and $\vec{r}$ tail to tail and $\hat{n}$ is a unit vector $\perp$ to the plane of $\vec{F}$ and $\vec{r}$ using a right handed system.
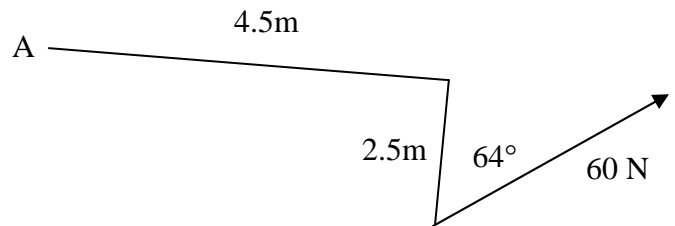
Ex. Find the magnitude of the torque about A(-3, 4, 5) of a force $\vec{f} = (3,1,-2)$ acting at B(-1, 1, 2). Assume units are in meters.
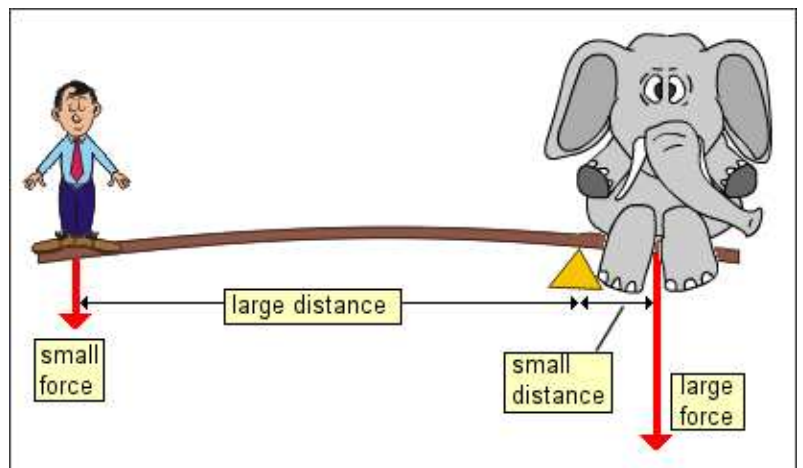
Homework for cross product applications

1. Find the magnitude of the moment about the point A:

4.5m

A

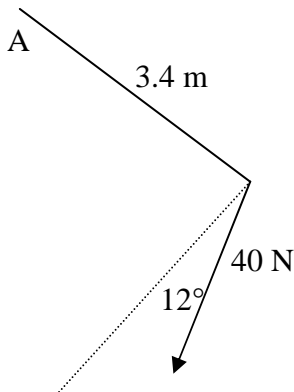2.5m   64°   60 N

2. Find the volume of the parallelepiped with the vectors $\{31\ u^3\}$

$\vec{a} = (-1,4,3), \vec{b} = (3,-3,1)\ and\ \vec{c} = (4,-2,7)$ as the vectors that generate the parallelepiped.

3. Find the magnitude of the moment about the point A:

A

3.4 m

40 N

12°

$\{133\ J\}$

4. An 85 N force is applied to the end of a 40 cm wrench and makes an angle of 55° with the handle of the wrench. What is the magnitude of the torque on the bolt at the other end of the wrench. What is the maximum torque that can be exerted by a 75 N force on this wrench and how can it be achieved? $\{27.9\ J,\ 30\ J\}$

5. If $\overrightarrow{OA}, \overrightarrow{OB},\ and\ \overrightarrow{OC}$ are three edges of a parallelepiped where O is (0,0,0), A is (2,-5,-3), B is (5,6,2), and C is (6,7,-2), find the coordinates of the other vertices of the parallelepiped and find the volume of the parallelepiped.   {other vertices (7,1,-1), (8,2,-5), (11, 13, 0), (13, 8, -3), $V = 159u^3$ }