Challenge Set #2 - Solutions

3. $l_1 : \frac{x-4}{5} = y+2 = \frac{z-3}{-4}$

a) Test $A(14, 0, -5)$ on l_1

$\therefore \frac{14-4}{5} = 0+2 = \frac{-5-3}{-4}$

$2 = 2 = 2$

$\therefore A \in l_1$

b) Answers vary.

$l_2 : (x, y, z) = (14, 0, -5) + t(d_1, d_2, d_3)$

where (d_1, d_2, d_3) is any direction vector that is not a scalar multiple of $(5, 1, -4)$

$\therefore$ let $\vec{d}_2 = (1, 1, -1)$ for example

3.c) l_1:

$x = 4 + 5m$

$y = -2 + m$

$z = 3 - 4m$

l_2:

$x = 14 + t$

$y = t$

$z = -5 - t$

Set x, y, z equal to each other & solve for t and m. If there is a unique t, m that satisfy l_1 and l_2, then they can be used to find the POI.

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l_1: $x = 2t$
$y = 3 + t$
$z = 1 + t$

has direction vector $\vec{d} = (2, 1, 1)$

l_2: $x = 6 - 3m$
$y = -4 + 2m$
$z = 18 + m$

has a z-intercept of $(0, 0, z)$

$\therefore 0 = 6 - 3m$

so $m = 2$

$0 = -4 + 2m$

so $m = 2$

$\therefore z = 18 + (2)$
$= 20$

and the z-int is $(0, 0, 20)$

l_3: $(x, y, z) = (0, 0, 20) + s(2, 1, 1)$

5. Scalar equations are defined by the unique normal vector of the line. In $\mathbb{R}^3$, a vector does not have a unique normal $\therefore$ No scalar eq.

6. $A(-9,-3,-16)$
$B(6,2,14)$
$l: (x,y,z) = (0,0,2) + t(3,1,6)$

1) Could test A, B on line l by subbing in and solving for unique parameter t

2) Could create direction vector $\vec{AB}$ and verify if $\vec{AB}$ is scalar multiple of $(3,1,6)$ ~~then~~
then create vector $\vec{AC}$ when $C:(0,0,2)$ and verify $\vec{AC}$ is scalar multple of $(3,1,6)$

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l_1

$x = 1 + k$
$y = 2 - k$
$z = -3 + k$

l_2

$x = 5 + 2t$
$y = -2 - 2t$
$z = -3 - 2t$

If they intersect there is a k, t so that x, y, z equal each other

So $1 + k = 5 + 2t \longrightarrow k - 2t = 4$ ①

$2 - k = -2 - 2t \longrightarrow -k + 2t = -4$ ②

$-3 + k = -3 - 2t \longrightarrow k + 2t = 0$ ③

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② + ③ $4t = -4$

$t = -1$

$\therefore k = 2$

Test for component of x

$k - 2t = 4$

$2 - 2(-1) = 4$ $\therefore$ True

$4 = 4$

$\therefore$ POI is $(3, 0, -1)$