MCV4U – Day 1 What's your limit? Date: ________________

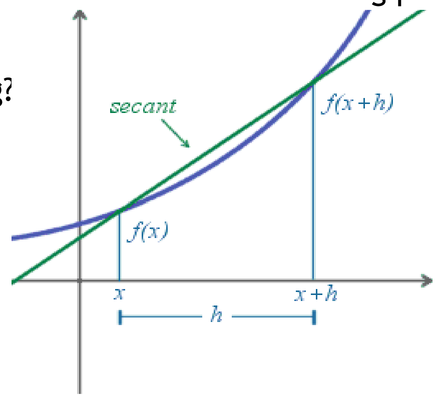
Recall: We used 1st and 2nd differences to help us find turning points and inflection points that in turn gave us intervals of increase, decrease and concavity.

The problem is that 1st differences are an average rate of change that we know to be only an approximation.

We eventually want to go from having an interval where we know there is a turning point to actually knowing exactly where that turning point is.

So how do we shrink the size of the interval we are using?

Recall the difference quotient: the slope of the secant.



Example 1 – Find the instantaneous rate of change of $f(x)=x^3-3x^2-x+3$ at $x=2$ by completing the following tables for preceding and following intervals. Avoid rounding too early.

Following Intervals	
IROC	h
	1
	0.1
	0.01
	0.001

Preceding Intervals	
IROC	h
	-1
	-0.1
	-0.01
	-0.001

Verify your answer using the Ti-83. Graph $f(x)$ and then draw a tangent line at $x=2$ by pressing [2nd], [prgm] and selecting 5: tangent.

1. What do you notice about your two columns of instantaneous rates of change?

2. What would we have to do to h to get the most accurate value for the IROC?

The notation we are using can be written as follows: $IROC = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

The Limit of a Function

The notation $\lim_{x \to a} f(x) = L$ is read "the limit of $f(x)$ as x approaches a is L."
It means that the value of the function $f(x)$ approaches the number L ($L \in \mathbf{R}$) as x approaches a from either side.

One-Sided Limits

Left-hand limit: $\lim_{x \to a^-} f(x)$ denotes the limit approaching a from the left side.

Right-hand limit: $\lim_{x \to a^+} f(x)$ denotes the limit approaching a from the right side.

Two-Sided Limit

If $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$, then $\lim_{x \to a} f(x)$ exists and is equal to L.
$\lim_{x \to a} f(x)$ is called a **two-sided limit**.

Limits That Fail to Exist

If $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, then $\lim_{x \to a} f(x)$ does not exist.

Some Basic Limits

If a and c are real numbers and n is an integer, then

1. $\lim_{x \to a} c = c$
2. $\lim_{x \to a} x = a$
3. $\lim_{x \to a} x^n = a^n$, provided $a \neq 0$ when $n \leq 0$

Limits of Polynomial Functions

For any polynomial function $P(x)$, $\lim_{x \to a} P(x) = P(a)$.

Problem Set A:

i) Sketch the given function and describe any key features, including the domain.
ii) Evaluate the limits indicated, if possible. If they are not possible, try to explain why.

Please make sure your solution is presentable should it be chosen to be shared.

For $f(x)=\frac{x-2}{x+3}$, find:

a) $\lim_{x\to 5} f(x)$ and IROC at $x=5$,
b) $\lim_{x\to -3} f(x)$ and IROC at $x=-3$,
c) $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

a)

x	f(x)
4	
4.9	
4.99	
4.999	

x	f(x)
6	
5.1	
5.01	
5.001	

h	$\frac{f(5+h)-f(5)}{h}$
-1	
-0.1	
-0.01	
-0.001	

h	$\frac{f(5+h)-f(5)}{h}$
1	
0.1	
0.01	
0.001	

Problem Set B:

i) Sketch the given function and describe any key features, including the domain.
ii) Evaluate the limits indicated, if possible. If they are not possible, try to explain why.

Please make sure your solution is presentable should it be chosen to be shared.

For $f(x)=\dfrac{(x+1)^2}{(x-4)^2}$, find:

a) $\lim_{x\to 8} f(x)$ and IROC at $x=8$,
b) $\lim_{x\to 4} f(x)$ and IROC at $x=4$,
c) $\lim_{x\to \infty} f(x)$ and $\lim_{x\to -\infty} f(x)$

Problem Set C:

i) Sketch the given function and describe any key features, including the domain.
ii) Evaluate the limits indicated, if possible. If they are not possible, try to explain why.

Please make sure your solution is presentable should it be chosen to be shared.

For $f(x)=\dfrac{x^2-4x+3}{x-3}$, find:

a) $\lim_{x\to 5} f(x)$ and IROC at $x=5$,
b) $\lim_{x\to 3} f(x)$ and IROC at $x=3$,
c) $\lim_{x\to \infty} f(x)$ and $\lim_{x\to -\infty} f(x)$

MCV4U - Day 2 <u>What's Your Limit?</u> Date: ________________

The purpose of a **<u>limit</u>** is to analyze the behavior of a function near a particular point.

The value of a limit comes from when you are unable to substitute into a function at a particular point. Think end behavior, asymptotes, holes or other discontinuities. A simple polynomial, for example, has no anomalies so the limit of *f* at *a* is simply *f(a)*.

What about something like $f(x) = \frac{\sin x}{x}$?

Complete each of the following problem sets. Be prepared to share your solution.

i) Sketch the given function and describe any key features, including the domain.
ii) Evaluate the limits indicated, if possible. If they are not possible, try to explain why.

Set A	$f(x) = \lvert x+5 \rvert$	a) $\lim_{x \to -5} f(x)$ b) IROC at $x = -5$ c) $\lim_{x \to \infty} f(x)$, $\lim_{x \to -\infty} f(x)$
Set B	$f(x) = x - x^{\frac{1}{3}}$	a) $\lim_{x \to 0} f(x)$ b) IROC at $x = 0$ c) $\lim_{x \to \infty} f(x)$, $\lim_{x \to -\infty} f(x)$
Set C	$f(x) = x^{\frac{2}{3}}$	a) $\lim_{x \to 0} f(x)$ b) IROC at $x = 0$ c) $\lim_{x \to \infty} f(x)$, $\lim_{x \to -\infty} f(x)$
Set D	$f(x) = \frac{x}{\lvert x \rvert}$	a) $\lim_{x \to 0} f(x)$ b) IROC at $x = 0$ c) $\lim_{x \to \infty} f(x)$, $\lim_{x \to -\infty} f(x)$
Set E	$f(x) = \begin{cases} x^2 & x < 1 \\ -(x-2)^2 + 3 & x \geq 1 \end{cases}$	a) $\lim_{x \to 1} f(x)$ b) IROC at $x = 1$ c) $\lim_{x \to \infty} f(x)$, $\lim_{x \to -\infty} f(x)$

MCV4U - Day 3 **What's your Limit?** Date: ________________

Why do we need limits? Math has "black hole" scenarios (dividing by zero, going to infinity), and limits give us an estimate when we can't compute a result directly.

The limit wonders, "If you can see everything *except* a single value, what do you think is there?".

Solve the following limits using algebraic manipulation:

a) $\lim_{x\to 3} x^2 - 3x + 1$

b) $\lim_{x\to -2} \frac{x-3}{x+5}$

c) $\lim_{x\to -1} \frac{x^2 - x - 2}{x+1}$

d) $\lim_{x\to 0} \frac{\frac{1}{5+x} - \frac{1}{5}}{x}$

e) $\lim_{x\to 0} \frac{\sqrt{x+1} - 1}{x}$

f) $\lim_{x\to 0} \frac{(x+8)^{\frac{1}{3}} - 2}{x}$

h) $\lim_{x\to 2} \frac{|x-2|}{x-2}$

Practice:

a) $\lim_{x\to 1} \frac{x^2 - 1}{x-1}$

b) $\lim_{h\to 0} \frac{(3+h)^2 - 9}{h}$

c) $\lim_{t\to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$

d) $\lim_{x\to 2} \frac{x^2 + x - 6}{x^2 - 4}$

e) $\lim_{t\to 0} \frac{\sqrt{2-t} - \sqrt{2}}{t}$

f) $\lim_{x\to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2}$

g) $\lim_{x\to -3} \sqrt{\frac{x-3}{2x+4}}$

h) $\lim_{x\to 27} \frac{27 - x}{x^{\frac{1}{3}} - 3}$