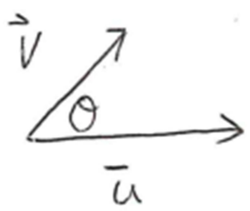
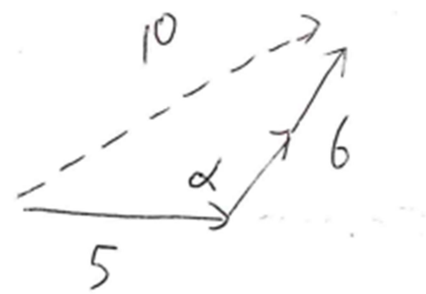
Challenge Set #3

1)





$$\alpha = 130.5$$

$$\therefore \theta = 49.5^\circ$$

$$\therefore (3\bar{u}+\bar{v})\cdot(\bar{u}-2\bar{v})$$

$$= 3\,\bar{u}\cdot\bar{u} - 5\bar{u}\cdot\bar{v} - 2\bar{v}\cdot\bar{v}$$

$$= 3(25) - 5(5)(3)\cos 49.5 - 2(9)$$

$$\doteq 8.3.$$

2)

a) For l_1, slope $= \frac{\text{rise}}{\text{run}} = \frac{-1}{1}$ $\rightsquigarrow$ Think down 1, right 1

$\therefore \bar{u} = (1, -1)$

b) For l_2, slope $= \frac{2}{1}$ $\rightsquigarrow$ Think up 2, right 1

$\therefore \bar{v} = (1, 2)$

c) $\therefore$ The angle between $\bar{u}$ and $\bar{v}$

$$\cos\theta = \frac{\bar{u} \cdot \bar{v}}{|\bar{u}||\bar{v}|}$$

$$= \frac{(1,-1)\cdot(1,2)}{\sqrt{2}\ \sqrt{5}}$$

$$= \frac{-1}{\sqrt{2}\sqrt{5}}$$

$$\theta = 108^\circ \text{ or } 72^\circ$$

3) Let $\bar{w} = \bar{u} \times \bar{v}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -1 \\ -1 & 2 & -1 \end{vmatrix}$$

$\bar{w} = (1, -1, -3)$

$$\therefore \hat{w} = \frac{1}{\sqrt{11}}(1, -1, -3)$$

$$= \left(\frac{1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}\right)$$

4) a) $\bar{a} \times \bar{b} = (-1, 2, -3)$

$\bar{a} \times \bar{c} = (-1, 2, -3)$ $\quad \therefore \bar{a} \times \bar{b} = \bar{a} \times \bar{c}$

b) We saw in part a) $\bar{b} \neq \bar{c}$ so it is not always true.

if $\bar{u} \times \bar{v} = \bar{u} \times \bar{w}$

then $\bar{u} \times \bar{v} - \bar{u} \times \bar{w} = 0$

$\bar{u} \times (\bar{v} - \bar{w}) = 0$

so $|\bar{u}| \, |\bar{v} - \bar{w}| \sin\theta \, \hat{e} = 0$

Since $|\bar{u}| \neq 0$

we have $|\bar{v} - \bar{w}| = 0$

$\therefore \bar{v} = \bar{w}$

But we can also have $\sin\theta = 0 \Rightarrow \theta = 0$

$\therefore \bar{u}$ and $\bar{v} - \bar{w}$ are collinear.

5. $\bar{a} = (1, 2, -3)$

$\bar{b} = (0, 1, 2)$

$\bar{c} = (1, 1, 1)$

show $(\bar{a} \times \bar{b}) \times \bar{c} \neq \bar{a} \times (\bar{b} \times \bar{c})$

$LS = \bar{a} \times \bar{b} \times \bar{c}$

$= (7, -2, 1) \times (1, 1, 1)$

$= (-3, -6, 9)$

$RS = (1, 2, -3) \times (-1, 2, -1)$

$= (4, 4, 4)$

$LS \neq RS$

6. if $\bar{u} \cdot (\bar{v} \times \bar{w}) = 0$

we know $\bar{u} \perp \bar{v} \times \bar{w}$

but $\bar{v} \times \bar{w} \perp \bar{v}, \bar{w}$

$\therefore \bar{u}, \bar{v}, \bar{w}$ all lie in the same plane

7. We can say $\bar{a} \perp \bar{b}$ or $\bar{a} = \bar{b}$

If $\bar{a} \perp \bar{b}$

then $\text{proj}_{\bar{b}} \bar{a} = \text{proj}_{\bar{a}} \bar{b} = \bar{0}$

If $\bar{a} = \bar{b}$

$\text{prob}_{\bar{b}} \bar{a} = \bar{b}$ and $\text{proj}_{\bar{a}} \bar{b} = \bar{a}$

$\therefore \text{proj}_{\bar{b}} \bar{a} = \text{proj}_{\bar{a}} \bar{b}$

8. Let $\bar{u} = (1,3,2)$ $\bar{v} = (5,0,-1)$ and $\bar{w} = (-4,3,3)$

Two non-collinear vectors are alway coplanar so

$\bar{v}$ & $\bar{w}$ lie in the same plane.

$\therefore \bar{v} \times \bar{w} \perp \bar{v}, \bar{w}$

If $\bar{u}$ is also in same plane are $\bar{v}, \bar{w}$

then $\bar{u} \perp \bar{v} \times \bar{w}$ as well

andso $\bar{u} \cdot (\bar{v} \times \bar{w}) = 0$

$\bar{u} \cdot (\bar{v} \times \bar{w})$

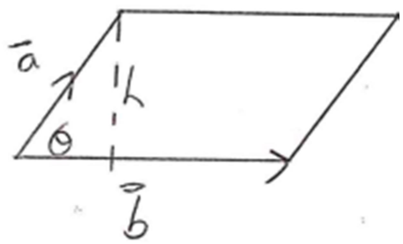
$= (1,3,2) \cdot (3,-11,15)$

$= 3 - 33 + 30$

$= 0$

$\therefore \bar{u}, \bar{v}, \bar{w}$ are coplanar

9)



$A =$ base $\times$ height

$= |\bar{b}|\, h$

$= |\bar{b}|\,|\bar{a}| \sin\theta$

$= |\bar{b} \times \bar{a}|$