MCV4U

Challenge Set #3

1. Given that $|\vec{u}| = 5$ and $|\vec{v}| = 3$ and $|\vec{u} + 2\vec{v}| = 10$, calculate $(3\vec{u} + \vec{v}) \cdot (\vec{u} - 2\vec{v})$.

2. Consider the two lines $\ell_1 : y = -x + 7$ and $\ell_2 : y = 2x + 3$.

   a. Determine a vector $\vec{u}$ which is parallel to ℓ_1.

   b. Determine a vector $\vec{v}$ which is parallel to ℓ_2.

   c. Find the angle θ between ℓ_1 and ℓ_2 by finding the angle between the two vectors you found in parts a) and b).

3. Given the vectors $\vec{u} = (-2, 1, -1)$ and $\vec{v} = (-1, 2, -1)$

   a. Find a unit vector perpendicular to both $\vec{u}$ and $\vec{v}$.

4. Given the vectors $\vec{a} = (1, 2, 1), \vec{b} = (2, 1, 0)$, and $\vec{c} = (4, 5, 2)$

   a. Show that $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$.

   b. For three vectors $\vec{u}, \vec{v}$, and $\vec{w}$, if $\vec{u} \times \vec{v} = \vec{u} \times \vec{w}$, must it be true that $\vec{v} = \vec{w}$?

5. Given the vectors $\vec{a} = (1, 2, -3), \vec{b} = (0, 1, 2)$, and $\vec{c} = (1, 1, 1)$, show that the cross product is not associative by verifying

$$\left(\vec{a} \times \vec{b}\right) \times \vec{c} \neq \vec{a} \times \left(\vec{b} \times \vec{c}\right)$$

6. For non-zero vectors $\vec{u}, \vec{v}$, and $\vec{w}$, under what geometric conditions will $\vec{u} \cdot (\vec{v} \times \vec{w})$ be equal to 0?

7. If the vector projection of a vector $\vec{a}$ on $\vec{b}$ is equal to the vector projection of vector $\vec{b}$ on $\vec{a}$, what can be said about the two vectors $\vec{a}$ and $\vec{b}$?

8. Vectors are said to be coplanar if they all lie in the same plane.
Determine if the vectors $(1, 3, 2), (5, 0, -1)$, and $(-4, 3, 3)$ are coplanar.

9. The area of a parallelogram is $A = base \times height$. Use vector concepts to develop another equation for area of a parallelogram.