Optimization

1) Economists define MARGINAL REVENUE as the rate of change of revenue with respect to the number of items produced. The Gadget Company determines that the revenue function for producing x gadgets is given by $R(x) = 200\sqrt{x^2+3}$
   a) What is the revenue if 50 gadgets are produced? {$10006}
   b) What is the marginal revenue at this level of production? {$199.88/item}

2) The function $P(t) = \dfrac{6}{1+6e^{-0.3t}}$ models a bacteria population in which the growth is initially rapid but eventually levels off. $P(t)$ is the mass of the culture in grams and t is the time in minutes.
   a) What is the initial mass of the culture? {0.857 grams}
   b) What is the average rate of change of the mass of the culture from $t = 5$ minutes to $t = 10$ minutes.
   c) What is the instantaneous rate of change of the mass of the culture after 12 minutes?

3) A rectangular rose garden will be surrounded on 3 sides by a brick wall and by a fence on the fourth side. The area of the garden will be 1000 m^2. The cost of the brick wall is $192/m. The cost of the fencing is $48/m. Find the dimensions of the garden so that the cost of the materials is as small as possible. {40 m by 25 m}

4) A ship anchored 9 km offshore. Opposite a point 5 km further along the straight shore, another ship is anchored 3 km offshore. A boat from the second ship must drop a passenger on shore and then proceed to the first ship. What is the shortest distance that the boat must travel? {13 km}

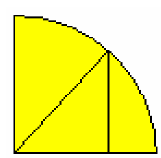
5) A steel storage tank for propane gas is to be constructed in the shape of a cylinder with a hemisphere on each end. If the volume is 100 m^3, what dimensions will require the least amount of steel? {sphere radius = 2.88 m}

6) At 3 pm, boat A leaves harbour and heads south at 6 km/h. Also at 3 pm, boat B is 30 km east of the harbour and is headed west at 8 km/h. Determine the minimum distance between the ships and the time when this occurs. {18 km at 5:24 pm}

7) Painters are painting the side of a building above a busy sidewalk. To protect pedestrians, a covered walkway 1.5 m wide and 3.2 m high is constructed along the wall. What is the length of the shortest ladder which will reach from the ground, over the walkway, to the side of the building? {6.5 m}

8) Find the rectangular solid of greatest volume that can be inscribed in a square pyramid of altitude 12 m and a square base of sides 18 m. {12 m by 12 m by 4m}

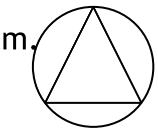
9) What is the maximum area of an inscribed right triangle formed in a quarter circle of radius $10\sqrt{2}$. {50 u^2}



10) A farmer is located 12 km from the nearest point of a straight railway. The railway company agrees to put in a siding at any place the farmer designates, and to haul his produce from there to the town for 5¢ per ton per km. The town is 80 km from the point nearest him. If he can haul by wagon for 13 ¢ per ton per km on a straight road to the siding, where should he ask to have the siding placed in order to minimize transportation costs? {75 km from town}

11) The publisher of a new edition of a popular cookbook knows that x thousand copies will sell if the price is
$p(x) = 10 - \frac{\sqrt{x}}{6}$. What price will maximize revenue? {$3.33}

12) Find the triangle of optimal area which can be inscribed in a circle of radius 10 cm.



13) An electric utility is required to run a cable from a transformer station on the shore of a lake to an island. The island is 6 km from the shore and the station is 12 km down the shoreline from a point opposite the island. It costs $4000/km to run cable on land, and $5000/km underwater. Find the path the cable should take for minimum cost of installation. What is the minimum cost?
{4 km along shore and then 10 km to the island, cost $66000}

14) An opened topped cylindrical container has a volume of $24\pi\ cm^3$. If the cost of the material used for the bottom is three times the cost of the material used for the curved part, find the dimensions that will minimize the cost.
$\{r = 2\ cm, \quad h = 6\ cm\}$

15) Ship A sailing south at 12 km/h is 45 km due north of ship B which is sailing west at 9 km/h. At what time is the distance between the ships a minimum? What is the minimum distance?

16) A convenience store currently sells 240 bags of milk weekly at a price of $3.50 each. Sales predictions indicate that each $0.25 decrease in price will increase sales by 60 bags weekly. If the store pays $2 for each bag of milk:
a) what price will maximize profit? {$3.25}
b) what price will maximize revenue? {$2.25}

17) A ladder is to be moved horizontally around a corner from a hallway 3 m wide to a hallway 2 m wide. Determine the length of the longest ladder that can negotiate the turn. Neglect the thickness of the ladder. {7.024 m}

18) Daniel makes a candleholder by inscribing a circular cylinder in a sphere of radius 10 cm. Find the dimensions of the cylinder which will optimize its volume. {h = 11.55 cm, r = 8.2 cm}

19) A calculus teacher can estimate the mathematical knowledge, *K*, that an average student retains over *t* months using $K(t) = 100 - 15\ln t$. In this equation *K* is measured in percent.
a) How much knowledge does an average student retain over two months?
b) How much knowledge does an average student retain after one year?
c) Find the instantaneous rate of change of the retention of mathematical knowledge at $t = 6, 12, 24$ months.

20) A sector of a circle whose radius is *r* and whose angle is θ has a fixed perimeter *P*. Find the values of *r* and θ so that the area of the sector is a maximum.
$\{r = \frac{P}{4}, \theta = 2\ \ radians\}$

21) Daniel makes a candleholder by inscribing a cylinder in a right circular cone. The height of the cone is 15 cm. The radius is 5 cm. Find the dimensions of the cylinder that will maximize its surface area, assuming that the dimensions remain proportional.