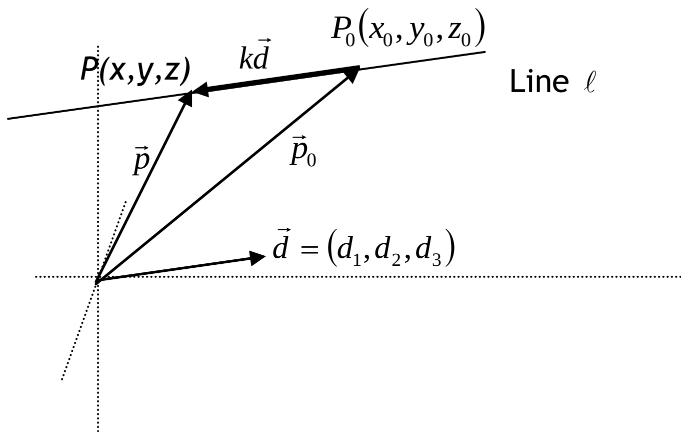
## Lines in $R^3$

Definition: A direction vector for a line is a vector parallel to the line. A line with direction vector $\vec{d} = (d_1, d_2, d_3)$.




$$P_0P = k\vec{d}$$
$$(x - x_0, y - y_0, z - z_0) = k(d_1, d_2, d_3)$$
$$(x, y, z) = (x_0, y_0, z_0) + k(d_1, d_2, d_3)$$
$$\overrightarrow{OP} = \overrightarrow{OP_0} + k\vec{d}$$
$$\vec{p} = \vec{p}_0 + k\vec{d}$$

These last three equations are called the vector equation of a line in space (there are an infinite number of representations). You could use any $\vec{d}$ and any $\vec{p}_0$. The scalar *k* is called a parameter.

If you equate the components, we get:

$$\begin{cases} x = x_0 + kd_1 \\ y = y_0 + kd_2 \\ z = z_0 + kd_3 \end{cases}$$

which is called the parametric equations of a line in three space.

If we then solve for the parameter, we get:

$$\frac{x - x_0}{d_1} = \frac{y - y_0}{d_2} = \frac{z - z_0}{d_3}$$

which is called the symmetric equation of a line in three space.

Definition: A normal vector for a line is a vector $\vec{n} = (n_1, n_2, n_3)$ which is perpendicular to the line.

So …. there is no scalar equation of a line (or Cartesian) in three space because there is no unique normal for a line in 3-space.

Ex. Find vector, parametric, and symmetric equations of the line that passes through $P_1(2,4,0)$ and $P_2(5,0,7)$. Does the point Q(-4,12,-14) lie on that line?

Ex. Given $\ell_1 : \vec{r}_1 = (3,4,3) + k(2,-1,5)$ and $\ell_2 : r_2 = (-9,8,-6) + m(-6,3,-15)$. Do these represent the same line?

Ex. Find symmetric equations of the line through A(1,2,3) and B(2,-1,3).

Ex. Do the equations $\ell_1 : \frac{x-5}{2} = \frac{y+4}{3} = \frac{z+1}{-5}$ and $\ell_2 : \frac{x+1}{-4} = \frac{y-1}{-6} = \frac{z+3}{10}$ represent the same line?