Intersection of a line and a plane

Substitute the line in parametric form into the scalar equation of the plane and solve for the parameter. Substitute this value of the parameter back into the equation of the line to find the point of intersection.

There are three possibilities:

Line // Plane

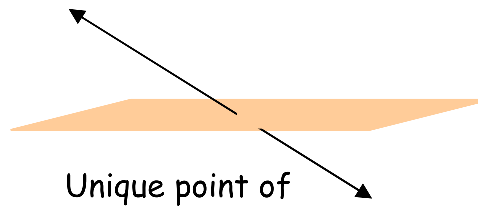
$$\vec{d}_{line} \bullet \vec{n}_{plane} = 0$$

$$0t = 3$$

Plane contains Line

$$\vec{d}_{line} \bullet \vec{n}_{plane} = 0$$

$$0t = 0$$



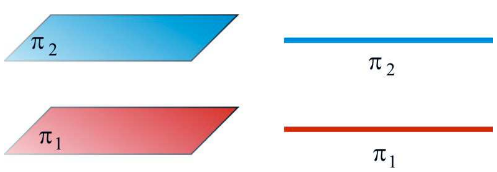
Unique point of intersection

$$\vec{d}_{line} \bullet \vec{n}_{plane} \neq 0$$

$$t = 4$$

Intersection of Two Planes

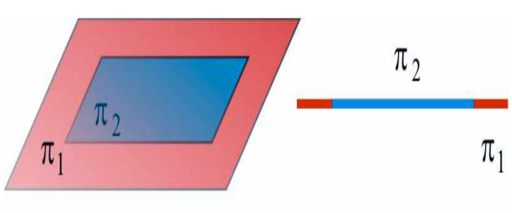
Using the two scalar equations eliminate one variable and then using the two scalar equations eliminate a different variable. You should have two equations with one common variable in both of them. Let this be your parameter and generate the equations of the line of intersection.
Two equations, three variables, you will not get one solution. Solve for x in terms of z then solve for y in terms of z and set z equal to a variable (parameter). This will give you three parametric equations which generate the equations of a line.



Planes are // $\therefore$ no intersection

normals are parallel

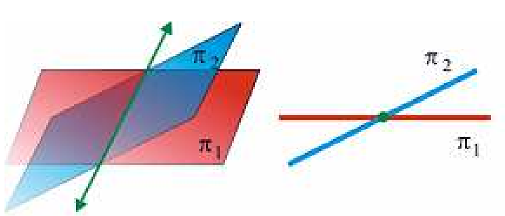
$$0 = 3$$



Planes are coincident $\therefore$ $\infty' ly$ many solutions

normals are parallel, they share a point

$$0 = 0$$

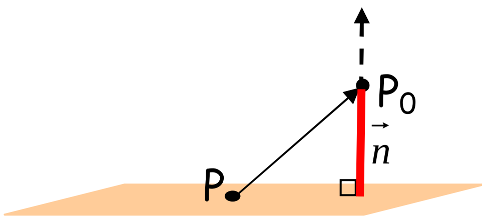


Intersect at a line

normals are not parallel.
Find equation of the line.

Distance between a point and a plane

Find the magnitude of the projection of the vector between the point and a point on the plane onto the normal for the plane. This will be the distance between the point and the plane. This technique can be used to find the distance between a line and a plane and the distance between parallel planes.



$$\boxed{\begin{aligned} dist &= \left|\mathrm{Proj}_{\vec{n}}\,\overrightarrow{PP_0}\right| \\ &= \left|\frac{\vec{n} \bullet \overrightarrow{PP_0}}{|\vec{n}|}\right| \end{aligned}}$$

Examples:

1) Find the intersection of the line $\vec{r} = (2,3,10) + t(1,1,-3)$ and the plane $2x + 3y - z + 5 = 0$.

(1,2,13)

2) Find the intersection of the line $\frac{x-2}{3} = \frac{y-1}{-1} = \frac{z}{3}$ and the plane $2x + 3y - z + 5 = 0$.

3) Find the intersection of the line $\vec{r} = (1,2,13) + t(2,0,4)$ and the plane $2x + 3y - z + 5 = 0$.

$(k, 2, 11+2k) \quad k \in \Re$

4) Find the intersection of the planes $x - 2y + z - 3 = 0$ and $2x + y - z + 1 = 0$

$$\begin{aligned} x &= \frac{2}{3} + \frac{1}{3}t \\ y &= t \\ z &= \frac{7}{3} + \frac{5}{3}t \end{aligned} \quad \text{or} \quad \begin{aligned} x &= \frac{2}{3} + t \\ y &= 3t \\ z &= \frac{7}{3} + 5t \end{aligned}$$

5) Find the distance from the point (-1,2,-3) to the plane $5x + y - 4z + 1 = 0$ 1.54