Challenge Set #1

5) Let the line be $l : (x, y) = (7, 3) + k(2, -5)$

Test $(5, 8)$

$\therefore (5, 8) = (7, 3) + k(2, -5)$

is true if $k = -1$

Test $(17, -22)$

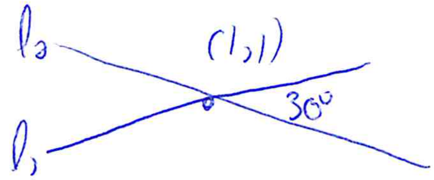
$\therefore (17, -22) = (7, 3) + k(2, -5)$

is true if $k = 5$

$\therefore$ Both points are on line l

6. Possible solution - Answers vary

Let the lines be l_1, l_2 and intersect at random pt $(1,1)$



$\therefore$ $l_1 : (x,y) = (1,1) + k(x_1, y_1)$

$l_2 : (x,y) = (1,1) + t(x_2, y_2)$

Let direction vector $\bar{d}_1 = (2,3)$
Let direction vector $\bar{d}_2 = (-1, y)$ $\Big\}$ anything

$$\therefore \bar{d}_1 \cdot \bar{d}_2 = |\bar{d}_1| |\bar{d}_2| \cos 30$$

$$-2 + 3y = \sqrt{13}\sqrt{1+y^2} \cos 30$$

$$-2 + 3y = \sqrt{13}\sqrt{1+y^2}\,\frac{\sqrt{3}}{2}$$

$$4 - 12y + 9y^2 = \frac{(13)(1+y^2)(3)}{4}$$

$$16 - 48y + 36y^2 = 39 + 39y^2$$

$$0 = 3y^2 + 48y + 23$$

$$\therefore y = \frac{-48 \pm \sqrt{2028}}{6} = \frac{-48 \pm 26\sqrt{3}}{6} = \frac{-24 \pm 13\sqrt{3}}{3}$$

Randomness of values can lead to messy solution

9)

$$(x, y, z) = (x_0, y_0, z_0) + k(d_1, d_2, d_3)$$

$$\begin{cases} x = x_0 + d_1 k \\ y = y_0 + d_2 k \\ z = z_0 + d_3 k \end{cases}$$

$$\frac{x - x_0}{d_1} = \frac{y - y_0}{d_2} = \frac{z - z_0}{d_3}$$

No standard form as a line in 3-space has no unique normal.

10 Let $l_1: 2x - 5y - 7 = 0$

$l_2: (x, y) = (-1, -2) + t(1, 3)$

Convert l_1 into vector form

$\therefore \bar{d}_1 = (5, 2)$

A point on l_1 is $(1, -1)$

$\therefore l_1: (x, y) = (1, -1) + k(5, 2)$

Intersections occur at common pts

sorry, #'s are not nice.

$\therefore$ l_1 l_2

$x = 1 + 5k$ $x = -1 + t$

$\therefore 1 + 5k = -1 + t$

$5k - t = -2$

$y = -1 + 2k$ $y = -2 + 3t$

$\therefore -1 + 2k = -2 + 3t$

$2k - 3t = -1$

solving this system $k = -0.385$

$t = 0.0769$

↳ sub in to find Point

$\therefore$ POI is when $t = 0.0769$

$k = -0.385$