- Recall that the instantaneous rate of change for a function $f(x)$ for any value of x is given by $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
- To distinguish the function $f(x)$ from the instantaneous rate of change function, the symbol we will use to describe the instantaneous rate of change function will be $f'(x)$. *We say" f prime of x", also known as" the derivative of $f(x)$".*
- So, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ which represents a function that is the rate of change of $f(x)$ for any value of x, or it represents the value of the slope of the tangent for $f(x)$ at any value of x.

Differentiability

A function $f(x)$ is said to be differentiable at *a* if $f'(a)$ exists. At points where *f* is not differentiable, we say that the derivative does not exist. Three ways for a derivative to fail to exist: cusps or corners, vertical tangents or discontinuities.

Continuity

A function $f(x)$ is said to be continuous at $x=a$ if $\lim_{x\to a} f(x)= f(a)$
A function is said to be continuous on the interval [a,b] if it is continuous at each point in the interval

A function may be continuous at a point but not differentiable.

<u>Exploring derivatives of polynomials</u>:

1) Try finding the derivative of each of the following polynomials from first principles. Divide the work amongst those in your group.

2) What seems to be a general rule for the derivative function of polynomials.

$f(x)$	$f'(x)$
$f(x)=5x+3$	
$f(x)=ax+b$	
$f(x)=x^2$	
$f(x)=3x^2-5$	
$f(x)=3x^2-2x-1$	
$f(x)=ax^2+bx+c$	
$f(x)=x^3$	
$f(x)=2x^3-6x$	
$f(x)=2x^3-\frac{1}{2}x^2-3x+5$	$f'(x)=6x^2-x-3$
$f(x)=x^4-10x^2+9$	$f'(x)=4x^3-20x$

3) Will this rule apply to the following functions with rational exponents? **<u>Check using first principles.</u>**

$f(x)=\sqrt{x}$ $\qquad g(x)=3x^{-5}$

4) Comment on the **<u>differentiability</u>** of polynomials.

Find the derivative of each function.

1. $y = x^8$

2. $y = \sqrt[3]{x}$

3. $y = x^{-\frac{2}{5}}$

4. $v(r) = \frac{4}{3}\pi r^3$

5. $y(t) = 6t^{-9}$

6. $f(x) = x^2 - 10x + 100$

7. $g(x) = x^{100} + 50x + 1$

8. $f(x) = (2x)^3$

9. $g(x) = x^2 + \frac{1}{x^2}$

10. $s(t) = t^8 + 6t^7 - 18t^2 + 2t$

11. $y = \frac{x^2 + 4x + 3}{x}$

12. $f(x) = x - 3x^{\frac{1}{3}}$

13. $y = \frac{3}{4x^3} + \frac{7}{2x^9} + \sqrt[5]{2x^4} - \sqrt[8]{3x^9}$

14. $y = 5x^{-4} - \frac{7}{8}x^{-2} + 3x^2 - 6$

15. $y = \frac{x^{12} - 2x^9 + 5x^{-7}}{4}$

16. Find an equation of the tangent line to the given curve at the specified point.

$y = x + \sqrt{x}$ at x = 1

17. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent is horizontal.