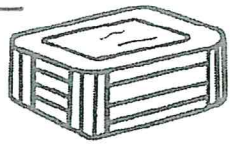
MCV4U

Investigating Rates of Change



Down the Drain:
The plug is pulled in a small hot tub. The table shows the volume of water in the tub from the moment the plug is pulled, until it is empty.

Draining Water from a Hot Tub			
Time (in 10 second intervals)	Volume (L)	$\Delta Volume$ (First Differences)	$\Delta^2 Volume$ (Second Differences)
0	1600		
		-256	
1	1344		23
		-233	
2	1111		22
		-211	
3	900		22
		-189	
4	711		23
		-167	
5	544		23
		-144	
6	400		22
		-122	
7	278		22
		-100	
8	178		22
		-78	
9	100		22
		-56	
10	44		23
		-33	
11	11		22
		-11	
12	0		

a) Calculate the first and second differences for the Volume.

b) Describe the trend in the Volume over the 120 seconds.

The volume is decreasing at a slower and slower rate

c) Determine the average rate of change from 10 seconds to 20 seconds.

$$AROC = \frac{1111 - 1344}{2 - 1}$$
$$= -233 \text{ L/s}$$

d) Determine the average rate of change from 100 seconds to 110 seconds. (interval 10 to 11)

$$AROC = \frac{11 - 44}{11 - 10}$$
$$= -33 \text{ L/s}$$

e) Describe the trend in the *rate of change* of the Volume over the 120 seconds.

The rate of change is increasing (becoming more positive)

f) Is there a connection between the first differences and the trend in the Volume?

The first differences are negative which indicates that the volume is decreasing.

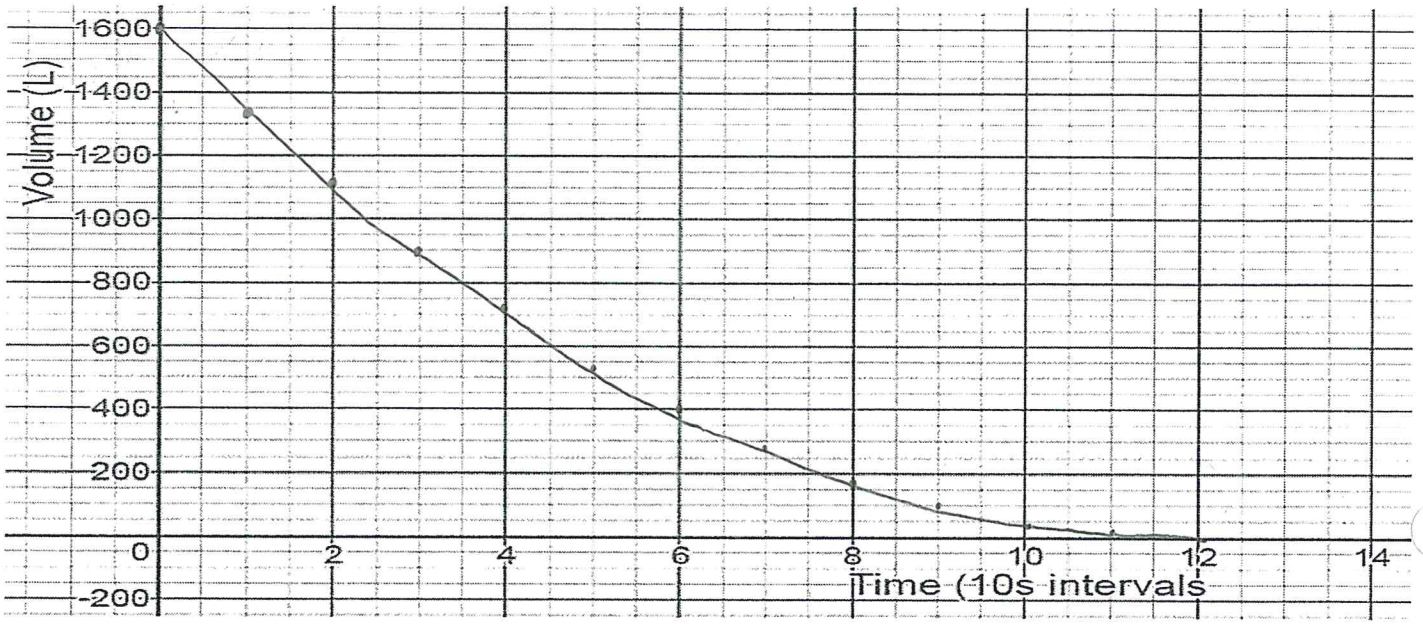
g) Is there a connection between the second differences and the trend in the *rate of change* of the Volume?

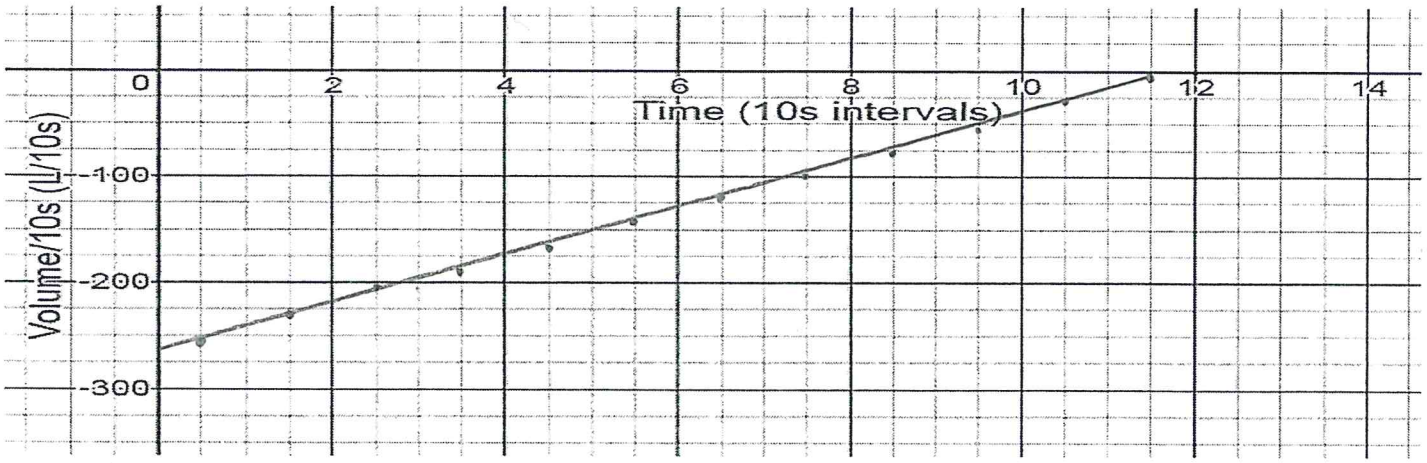
The second differences are positive which indicates that the rate of change (ΔV) is increasing.

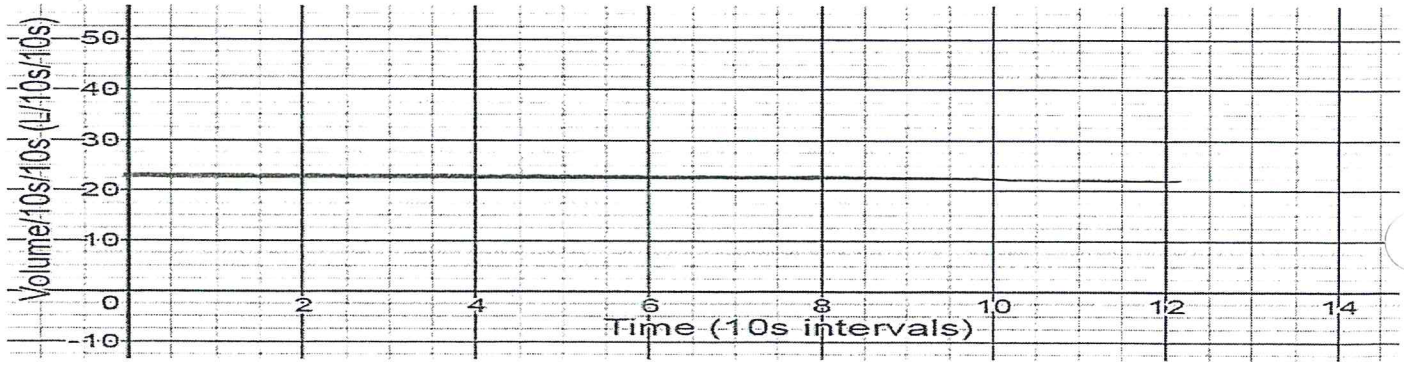
h) Using the grids below, graph the Volume, the first differences and the second differences over the 120 seconds.

i) Analyze the three graphs. Take note of any similarities/connections between the graphs.

V	ΔV	$\Delta^2 V$
quadratic	linear	constant
decreasing →	negative	
	increasing →	positive







Something Cubic:
The graph shows some object's height above some perceived "zero line" over some amount of time.

a) Calculate the first and second differences for height

b) Describe the trend in the Height over the 20 seconds.

increases, decreases, increases

c) Is there a connection between the first differences and the trend in the Height?

yes

positive, negative, positive

Height of Something That is Cubic			
Time	Height	ΔHeight (1st differences)	Δ^2 Height (2nd differences)
0	-34		
1	-10.8	23.2	-5.4
2	7	17.8	-3.8
3	20	14	-5.2
4	28.8	8.8	-3.6
5	34	5.2	-3
6	36.2	2.2	-2.4
7	36	-0.2	-1.8
8	34	-2	-1.2
9	30.8	-3.2	-0.6
10	27	-3.8	0
11	23.2	-3.8	+0.6
12	20	-3.2	+1.2
13	18	-2	+1.8
14	17.8	-0.2	+2.4
15	20	+2.2	+3
16	25.2	+5.2	+3.4
17	34	+8.6	+4.4
18	47	+13	+4.8
19	64.8	+17.8	+5.4
20	88	+23.2	

d) Determine the average rate of change from 1 second to 3 seconds.

$$AROC = \frac{20-(-10.8)}{3-1} = \frac{30.8}{2} = 15.4 \text{ m/s}$$

e) Determine the average rate of change from 5 seconds to 7 seconds.

$$AROC = \frac{36-34}{7-5} = \frac{2}{2} = 1 \text{ m/s}$$

f) Describe the trend in the *average rate of change* of the height over the 20 seconds.

decreases, increases

g) Is there a connection between the second differences and the trend in the *average rate of change* of the Height?

yes

negative, positive

h) Determine the instantaneous rate of change at 2 seconds (by drawing a tangent line and calculating the slope).

$$IROC = \frac{80-(-20)}{7-0} \doteq 14.3 \text{ m/s}$$

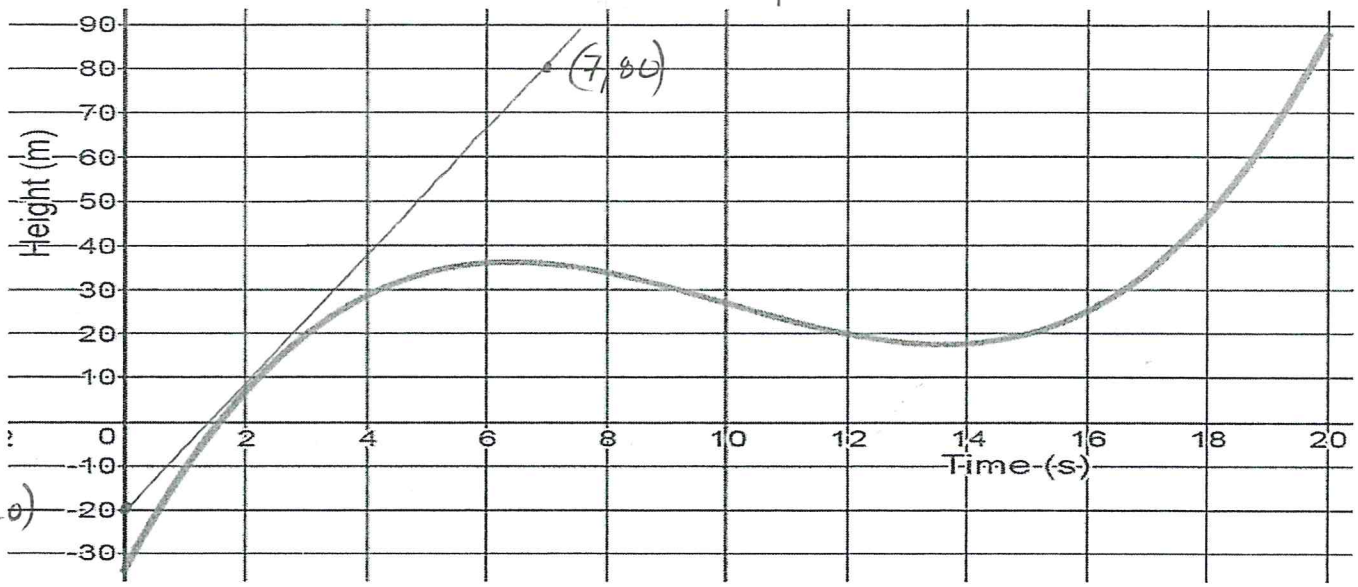
i) Using the grids below, graph the Height, the first differences and the second differences over the 20 seconds.

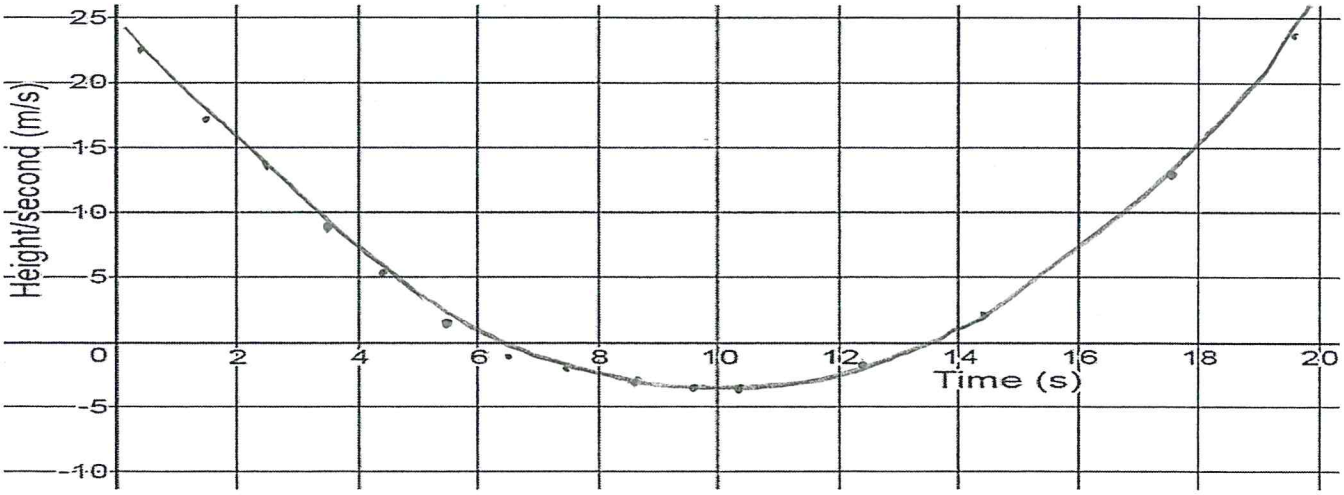
j) Analyze the three graphs. Take note of any similarities/connections between the graphs.

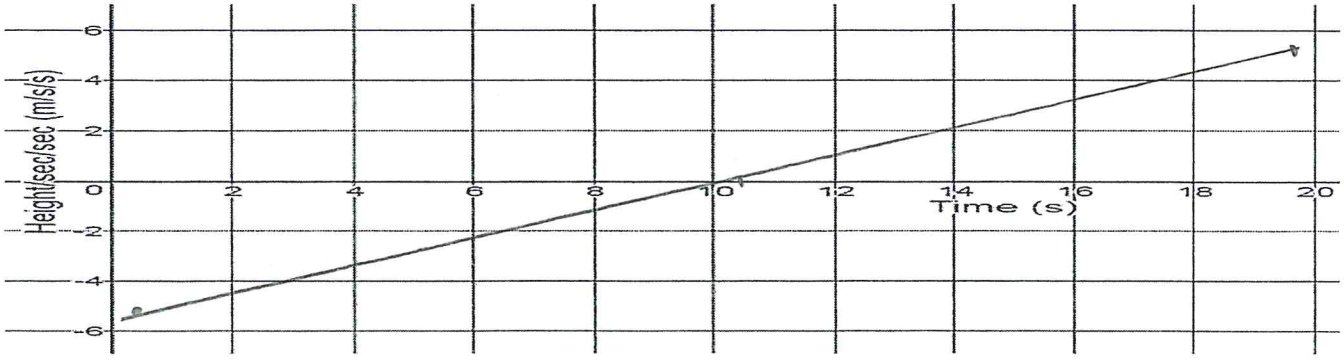
h	Δh	$\Delta^2 h$
cubic	quadratic	linear

increasing / turning point / dec → +ve / 0 / -ve

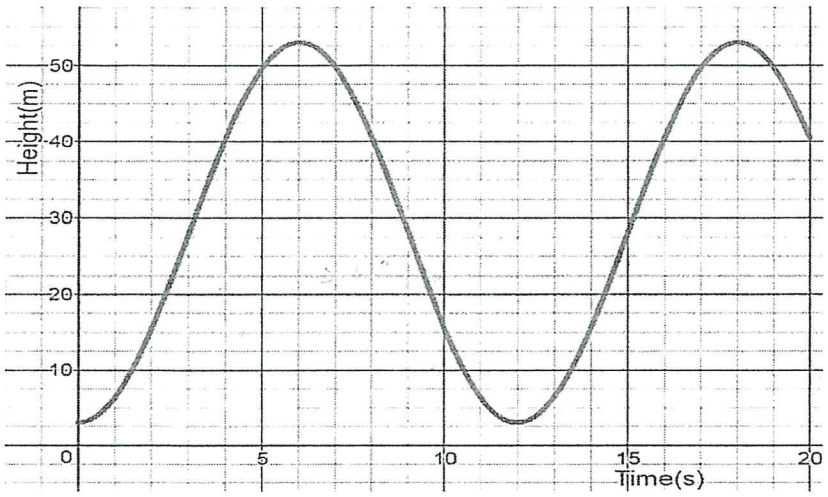
Concave up / inflection point / concave down → inc / TP / dec → +ve / 0 / -ve







Height of a Ferris wheel Rider over time



Height of Ferris Wheel			
Time (s)	Height (m)	Δ Height (1st diff)	Δ² Height (2nd diff)
0	3		
1	6.4	3.4	
2	15.5	9.1	5.7
3	28	12.5	3.4
4	40.5	12.5	0
5	49.7	9.2	-3.3
6	53	3.3	-5.9
7	49.7	-3.3	-6.6
8	40.5	-9.2	-5.9
9	28	-12.5	-3.3
10	15.5	-12.5	0
11	6.4	-9.1	3.4
12	3	-3.4	5.7
13	6.4	3.4	6.8
14	15.5	9.1	5.7
15	28	12.5	3.4
16	40.5	12.5	0
17	49.7	9.2	-3.3
18	53	3.3	-5.9
19	49.7	-3.3	-6.6
20	40.5	-9.2	-5.9

a) Calculate the first and second differences for the Height.

b) Describe the trend in the Height over the first 20 seconds.

increases, decreases, increases, decreases

c) Determine the average rate of change from 1 second to 3 seconds.

$$AROC = \frac{28-6.4}{3-1} = 10.8 \text{ m/s}$$

d) Determine the average rate of change from 2 seconds to 4 seconds.

$$AROC = \frac{40.5-15.5}{4-2} = 12.5 \text{ m/s}$$

e) Determine the average rate of change from 6 seconds to 8 seconds.

$$AROC = \frac{40.5-53}{8-6} = -6.25 \text{ m/s}$$

f) Describe the trend in the *average rate of change* of the Height over the first 20 seconds.

+ve, -ve, +ve, -ve

g) Is there a connection between the first differences and the trend in the Height?

first difference +ve then height increasing
" " -ve " " decreasing

h) Is there a connection between the second differences and the trend in the *average rate of change* of the Height?

second difference +ve then AROC increasing
" " -ve " " decreasing

i) Using the grids below, graph the Height, the first differences and the second differences over the 20 seconds.

j) Analyze the three graphs. Take note of any similarities/connections between the graphs.

h	Δh	$\Delta^2 h$
trig	trig	trig
inc/TP/dec →	+ve/0/-ve	
cu/IP/CD →	inc/TP/dec →	+ve/0/-ve