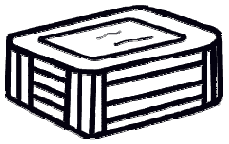
MCV4U

Investigating Rates of Change



Down the Drain:
The plug is pulled in a small hot tub. The table shows the volume of water in the tub from the moment the plug is pulled, until it is empty.

a) Calculate the first and second differences for the Volume.

Draining Water from a Hot Tub			
Time (in 10 second intervals)	Volume (L)	$\Delta Volume$ (First Differences)	$\Delta^2 Volume$ (Second Differences)
0	1600		
1	1344		
2	1111		
3	900		
4	711		
5	544		
6	400		
7	278		
8	178		
9	100		
10	44		
11	11		
12	0		

b) Describe the trend in the Volume over the 120 seconds.

c) Determine the average rate of change from 10 seconds to 20 seconds.

d) Determine the average rate of change from 100 seconds to 110 seconds.

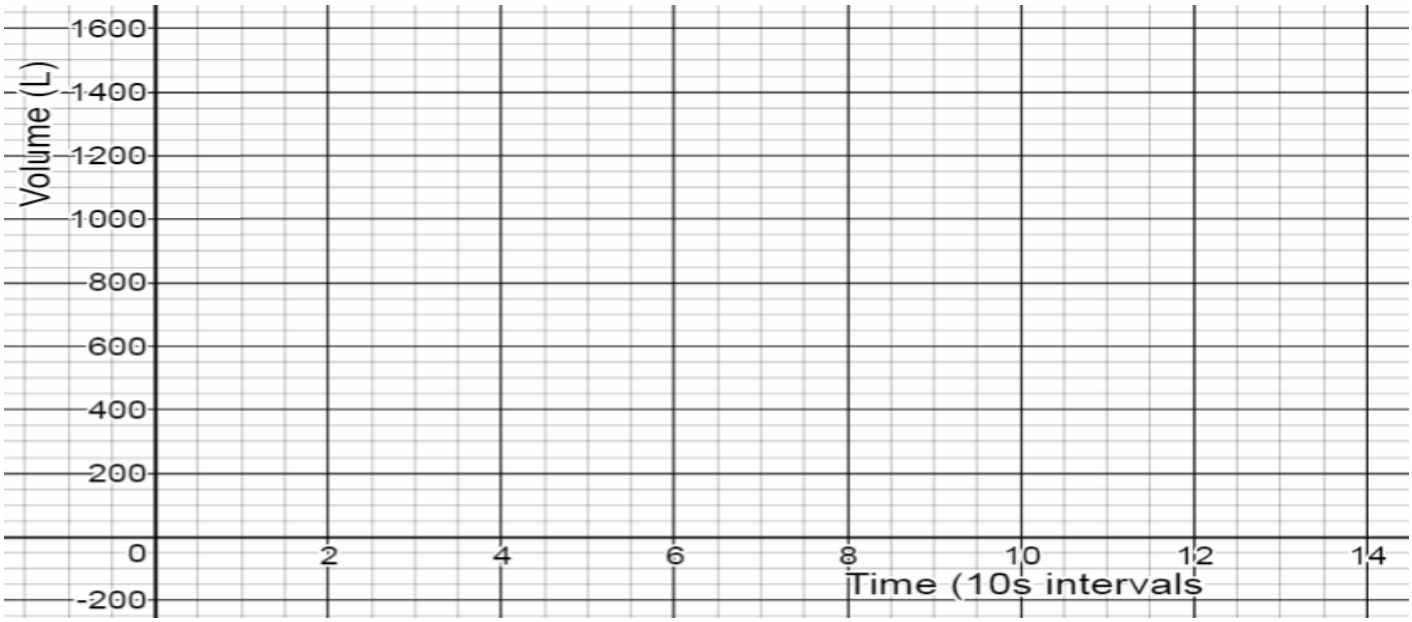
e) Describe the trend in the *rate of change* of the Volume over the 120 seconds.

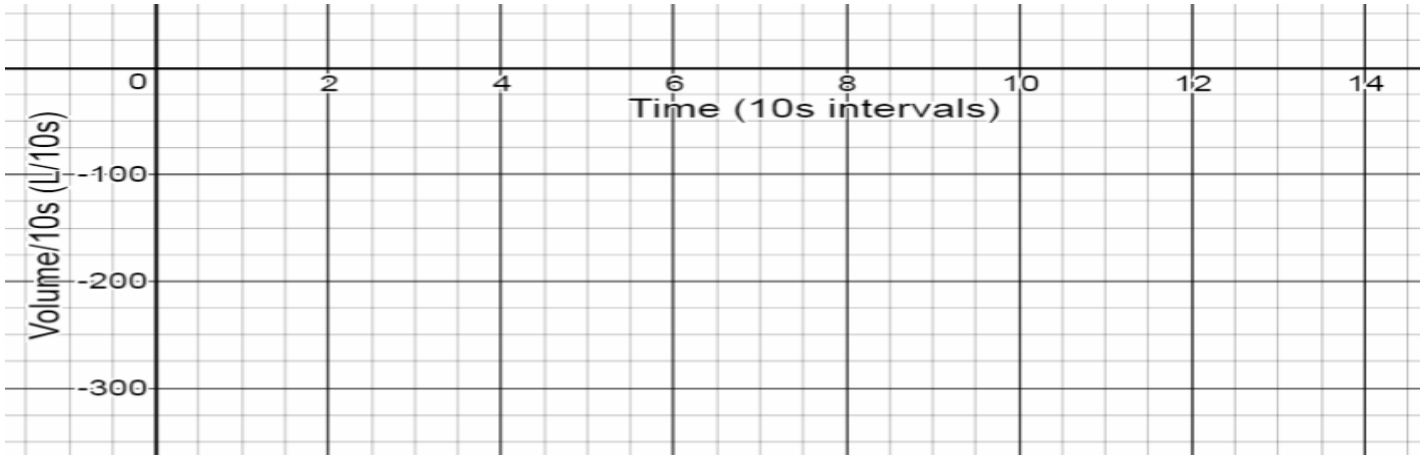
f) Is there a connection between the first differences and the trend in the Volume?

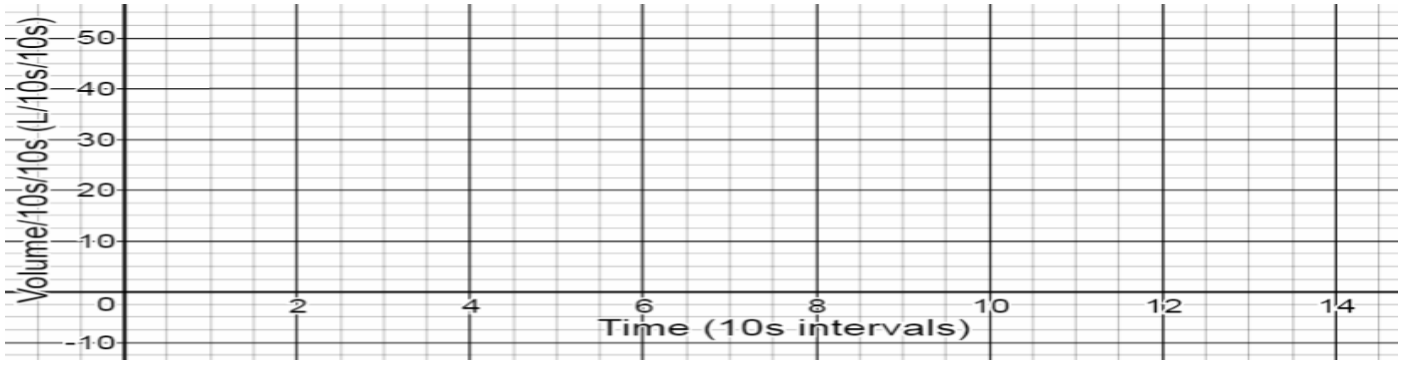
g) Is there a connection between the second differences and the trend in the *rate of change* of the Volume?

h) Using the grids below, graph the Volume, the first differences and the second differences over the 120 seconds.

i) Analyze the three graphs. Take note of any similarities/connections between the graphs.







Something Cubic:
The graph shows some object's height above some perceived "zero line" over some amount of time.

a) Calculate the first and second differences for height

b) Describe the trend in the Height over the 20 seconds.

c) Is there a connection between the first differences and the trend in the Height?

Height of Something That is Cubic			
Time	**Height**	$\Delta Height$ (1st differences)	$\Delta^2 Height$ (2nd differences)
0	-34		
1	-10.8		
2	7		
3	20		
4	28.8		
5	34		
6	36.2		
7	36		
8	34		
9	30.8		
10	27		
11	23.2		
12	20		
13	18		
14	17.8		
15	20		
16	25.2		
17	34		
18	47		
19	64.8		
20	88		

d) Determine the average rate of change from 1 second to 3 seconds.

e) Determine the average rate of change from 5 seconds to 7 seconds.

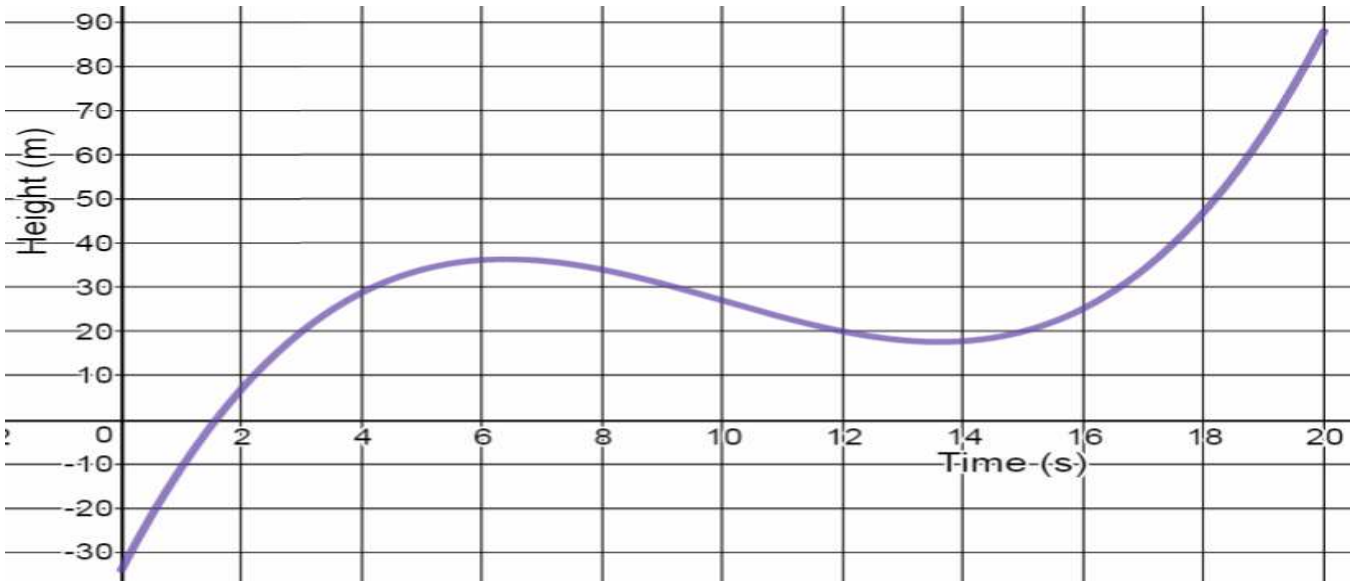
f) Describe the trend in the *average rate of change* of the height over the 20 seconds.

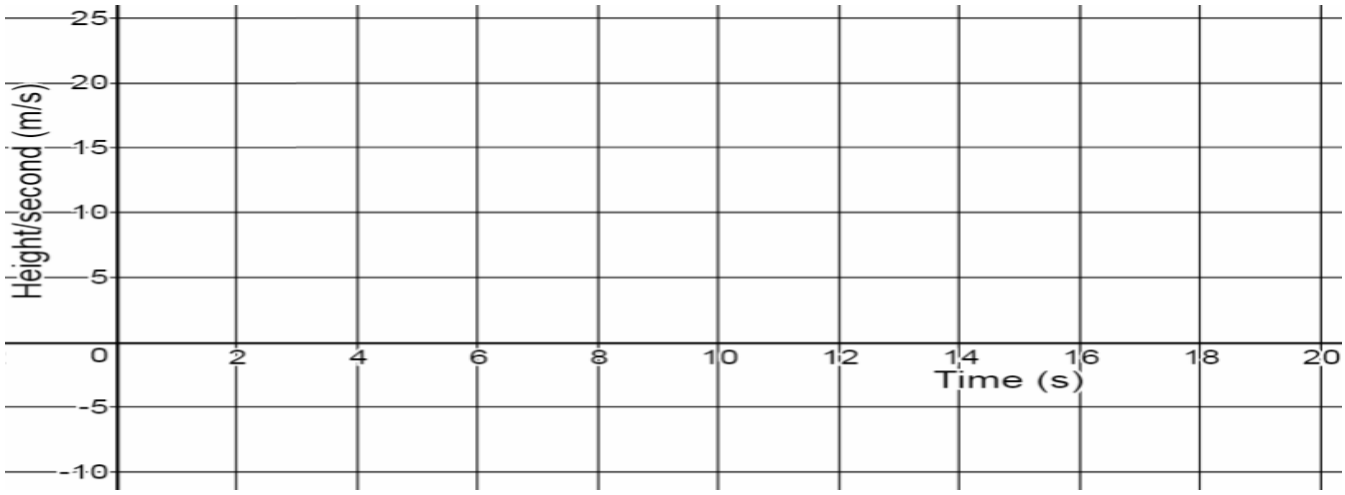
g) Is there a connection between the second differences and the trend in the *average rate of change* of the Height?

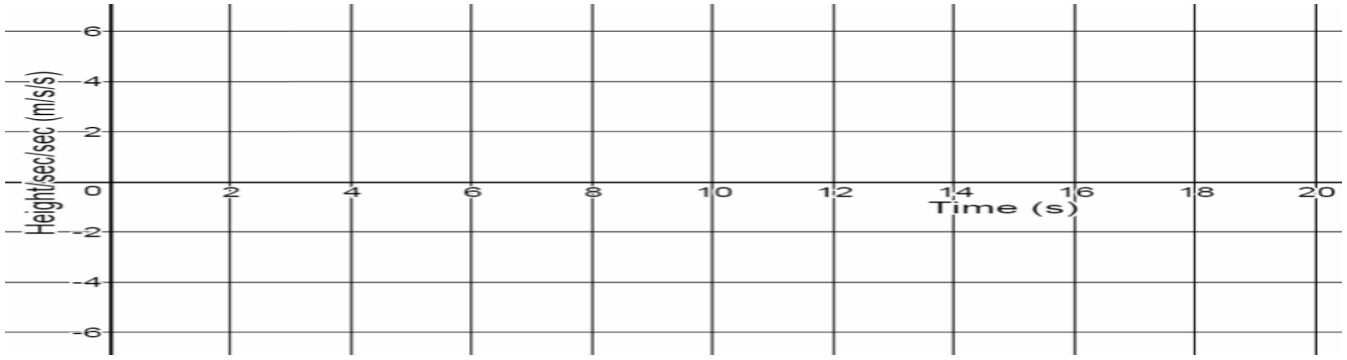
h) Determine the instantaneous rate of change at 2 seconds (by drawing a tangent line and calculating the slope).

i) Using the grids below, graph the Height, the first differences and the second differences over the 20 seconds.

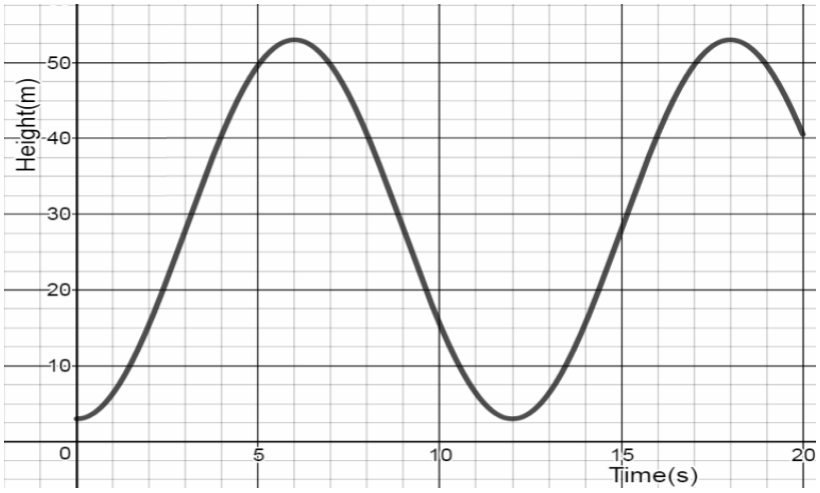
j) Analyze the three graphs. Take note of any similarities/connections between the graphs.







Height of a Ferris wheel Rider over time



Height of Ferris Wheel			
Time (s)	Height (m)	Δ Height (1^{st} diff)	Δ^2 Height (2^{nd} diff)
0	3		
1	6.4		
2	15.5		
3	28		
4	40.5		
5	49.7		
6	53		
7	49.7		
8	40.5		
9	28		
10	15.5		
11	6.4		
12	3		
13	6.4		
14	15.5		
15	28		
16	40.5		
17	49.7		
18	53		
19	49.7		
20	40.5		

a) Calculate the first and second differences for the Height.

b) Describe the trend in the Height over the first 20 seconds.

c) Determine the average rate of change from 1 second to 3 seconds.

d) Determine the average rate of change from 2 seconds to 4 seconds.

e) Determine the average rate of change from 6 seconds to 8 seconds.

f) Describe the trend in the *average rate of change* of the Height over the first 20 seconds.

g) Is there a connection between the first differences and the trend in the Height?

h) Is there a connection between the second differences and the trend in the *average rate of change* of the Height?

i) Using the grids below, graph the Height, the first differences and the second differences over the 20 seconds.

j) Analyze the three graphs. Take note of any similarities/connections between the graphs.