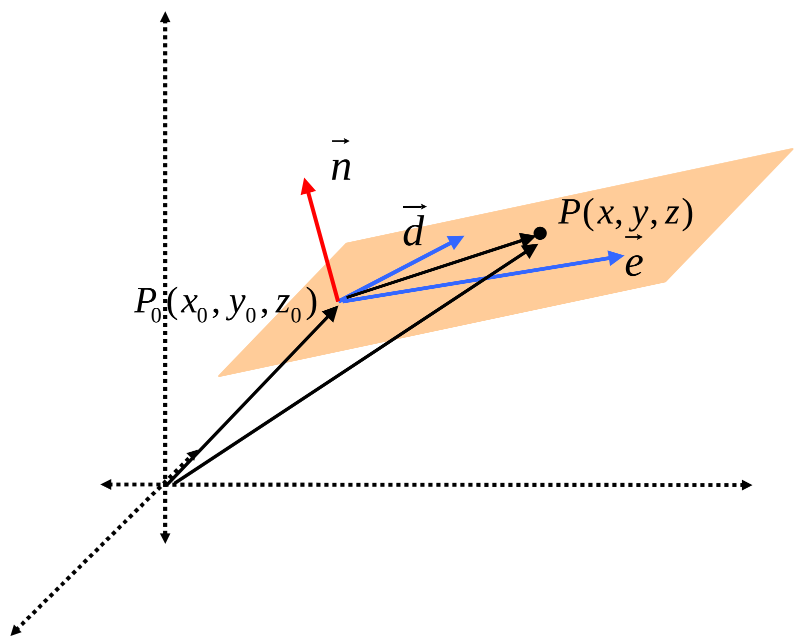
Equations of Planes

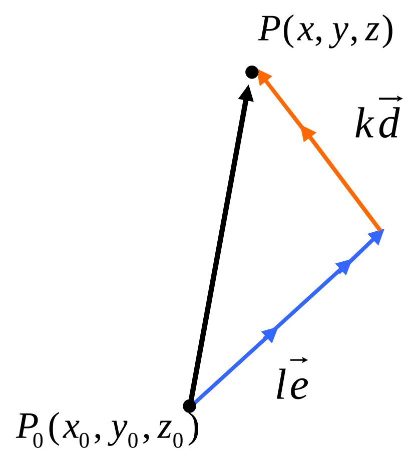
Minimum requirements to define a plane:
1) 3 non-collinear points
2) one point on the plane and two non-collinear direction vectors for the plane

$$\overrightarrow{OP} = \overrightarrow{OP_0} + \overrightarrow{P_0P}$$
$$\overrightarrow{OP} = \overrightarrow{OP_0} + k\vec{d} + l\vec{e}$$
$$(x, y, z) = (x_0, y_0, z_0) + k(d_1, d_2, d_3) + l(e_1, e_2, e_3)$$
$$\vec{p} = \vec{p}_0 + k\vec{d} + l\vec{e}$$

Vector Equation



Given direction vectors $\vec{d}$ and $\vec{e}$, the vector $\overrightarrow{P_0P}$ from a specific point P_0 to any general point P can be written as a linear combination of $\vec{d}$ and $\vec{e}$



So, $\overrightarrow{P_0P} = k\vec{d} + l\vec{e}$

These last three equations are called the vector equation of a plane in space (there are an infinite number of representations). You could use any $\vec{d}$ or $\vec{e}$ and any $\vec{p}_0$. The scalar *k* and *l* are called parameters.

If you equate the components, we get:

$$\begin{cases} x = x_0 + kd_1 + le_1 \\ y = y_0 + kd_2 + le_2 \\ z = z_0 + kd_3 + le_3 \end{cases}$$

which are called the parametric equations of a plane in three space.

Definition: A normal vector for a plane is a vector $\vec{n} = (n_1, n_2, n_3)$ which is perpendicular to the plane.

So…….

$\overrightarrow{P_0P} \perp \vec{n}$

$\overrightarrow{P_0P} \cdot \vec{n} = 0$

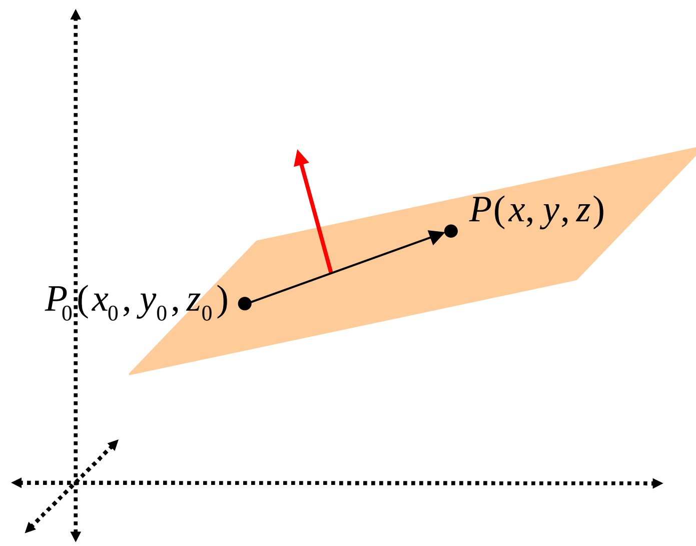
$(x - x_0, y - y_0, z - z_0) \cdot (n_1, n_2, n_3) = 0$

$n_1(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$

$n_1x + n_2y + n_3z + (-n_1x_0 - n_2y_0 - n_3z_0) = 0$

$n_1x + n_2y + n_3z + D = 0$

$Ax + By + Cz + D = 0$



So this last equation is the scalar equation of a plane (or Cartesian) in three space. Note that $\vec{n} = (A, B, C)$

Ex1. Find vector, parametric and scalar equations of the plane through A(1,1,2), B(-1,3,4), C(0, -1,5). Does Q(-1,1,3) lie on the plane?

Ex2. Given $\ell_1 : \vec{r_1} = (3,4,3) + k(2,-1,5)$ and $\ell_2 : r_2 = (-9,8,-6) + m(-6,3,-15)$, find the vector equation of the plane containing these two <u>parallel</u> lines.