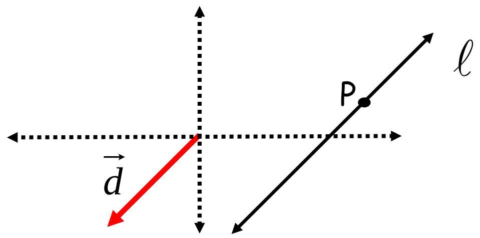
Lines in Two Space

Definition: A direction vector for a line is a vector parallel to the line. A line with direction vector $\vec{d} = (d_1, d_2)$ has slope $m = \frac{d_2}{d_1}$.

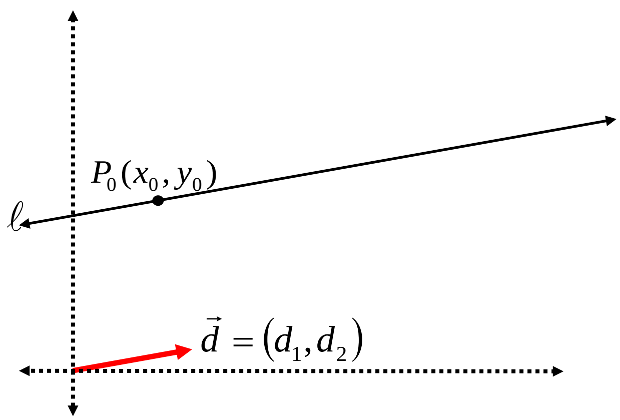
RECALL: Parallel (collinear) vectors can be written as scalar multiples of each other.

To get the equation of a line, we need two points (because two points make a direction vector) or a point and a direction vector.

Example – Sketch the line that passes through P(5,1) and has a direction vector $\vec{d} = (-3, -3)$.

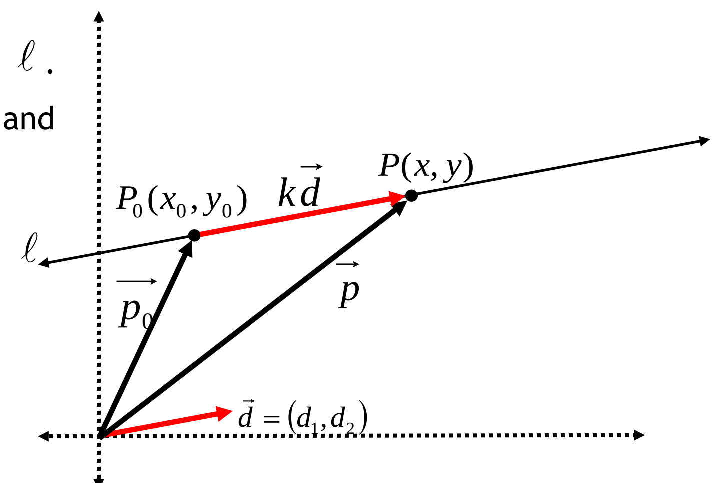


Suppose we have a line ℓ, with point $P_0(x_0, y_0)$ and direction $\vec{d} = (d_1, d_2)$



Now suppose $P(x, y)$ is a general point on ℓ.
The point P_0 has position vector $\overrightarrow{OP_0} = \overrightarrow{p_0}$ and the point P has position vector $\overrightarrow{OP} = \vec{p}$

We can write $\overrightarrow{P_0P} = k\vec{d}$, where $k \in \Re$, so that $k\vec{d}$ is any vector collinear with $\vec{d}$.



By the triangle law, we have

$$\overrightarrow{OP} = \overrightarrow{OP_0} + k\vec{d}$$
$$\vec{p} = \overrightarrow{p_0} + k\vec{d} \longrightarrow (x, y) = (x_0, y_0) + k(d_1, d_2)$$

These equations are called the <u>vector equations</u> of a line in the plane (there are an infinite number of representations). You could use any $\vec{d}$ and any $\vec{p}_0$. The scalar *k* is called a <u>parameter</u>.

If we equate the components, we get

$$\begin{cases} x = x_0 + kd_1 \\ y = y_0 + kd_2 \end{cases}$$

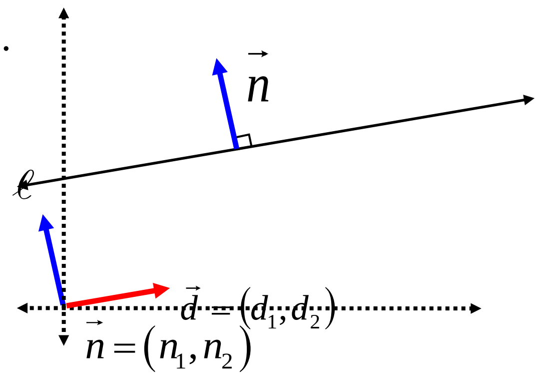
which is called the <u>parametric equations</u> of a line in two space.

If we then solve for the parameter, we get the <u>symmetric equation</u> of a line in two space.

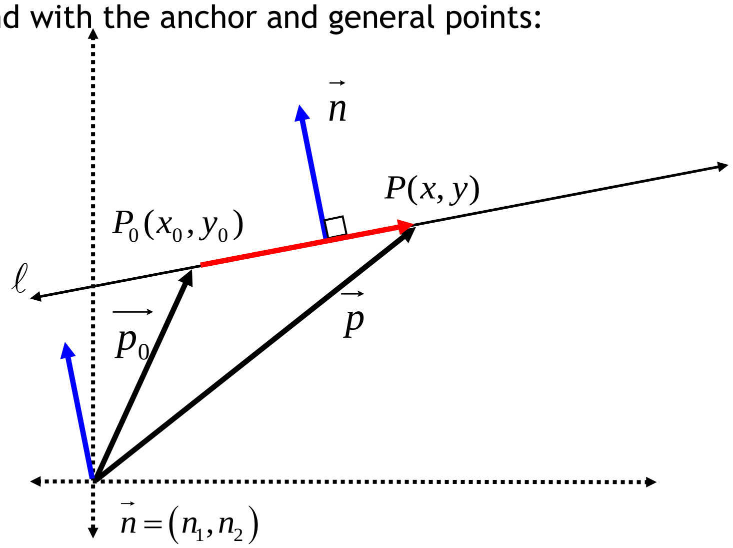
$x = x_0 + kd_1$ $k = \dfrac{x - x_0}{d_1}$	$y = y_0 + kd_2$ $k = \dfrac{y - y_0}{d_2}$	Now equate *k* = *k* $\therefore \dfrac{x - x_0}{d_1} = \dfrac{y - y_0}{d_2}$

Example – Determine the vector equation for the line that is perpendicular to $\vec{p} = (4,1) + k(-3,2),\ k \in \Re$, and passes through point P(6,5). $\{\vec{p} = (6,5) + k(2,3),\ k \in \Re\}$

Definition: A <u>normal vector</u> for a line is a vector $\vec{n} = (n_1, n_2)$ which is perpendicular to the line. A line with direction vector $\vec{d} = (d_1, d_2)$ has normal $\vec{n} = (d_2, -d_1)$.



And with the anchor and general points:



$\overrightarrow{P_0P} \perp \vec{n}$ $\overrightarrow{P_0P} \cdot \vec{n} = 0$ $(x - x_0, y - y_0) \cdot (n_1, n_2) = 0$ $n_1(x - x_0) + n_2(y - y_0) = 0$ $n_1x + n_2y + (-n_1x_0 - n_2y_0) = 0$ $n_1x + n_2y + C = 0$ $Ax + By + C = 0$	So this last equation is the <u>standard equation</u> of a line which is now called the scalar equation of a line (or <u>Cartesian</u>) in two space. Note that $\vec{n} = (A, B)$ $Ax + By + C = 0$