## MCV 4U 1: Representing Vectors

## Problems:

1. State whether each statement is true or false. Justify your decision.
a. If two vectors have the same magnitude, then they are equal.
b. If two vectors are equal, then they have the same magnitude.
c. If two vectors are parallel, then they are either equal or opposite vectors.
d. If two vectors have the same magnitude, then they are either equal or opposite vectors.
2. For each of the following, state whether the quantity is a scalar or a vector and give a brief explanation why: height, temperature, weight, mass, area, volume, distance, displacement, speed, force, and velocity.
3. Square $A B C D$ is drawn as shown below with the diagonals intersecting at $E$

a. State four pairs of equivalent vectors.
b. State four pairs of opposite vectors.
c. State two pairs of vectors whose magnitudes are equal but whose directions are perpendicular to each other.
4. Given the vector $\overrightarrow{A B}$ as shown, draw a vector
a. equal to $\overrightarrow{A B}$
b. opposite to $\overrightarrow{A B}$
c. whose magnitude equals $|\overrightarrow{A B}|$ but is not equal to $\overrightarrow{A B}$
d. whose magnitude is twice that of $\overrightarrow{A B}$ and in the same direction
e. whose magnitude is half that of $\overrightarrow{A B}$ and in the opposite direction

## Answers: Section 6.1, pp. 279-281

1. a. False; two vectors with the same magnitude can have different directions, so they are not equal.
b. True; equal vectors have the same direction and the same magnitude.
c. False; equal or opposite vectors must be parallel and have the same magnitude. If two parallel vectors have different magnitude, they cannot be equal or opposite.
d. False; equal or opposite vectors must be parallel and have the same magnitude. Two vectors with the same magnitude can have directions that are not parallel, so they are not equal or opposite.
2. The following are scalars: height, temperature, mass, area, volume, distance, and speed. There is not a direction associated with any of these qualities.
The following are vectors: weight, displacement, force, and velocity. There is a direction associated with each of these qualities.
3. Answers may vary. For example:
a. $\overrightarrow{A D}=\overrightarrow{B C} ; \overrightarrow{A B}=\overrightarrow{D C} ; \overrightarrow{A E}=\overrightarrow{E C}$; $\overrightarrow{D E}=\overrightarrow{E B}$
b. $\overrightarrow{A D}=-\overrightarrow{C B} ; \overrightarrow{A B}=-\overrightarrow{C D}$;
$\overrightarrow{A E}=-\overrightarrow{C E} ; \overrightarrow{E D}=-\overrightarrow{E B} ;$
$\overrightarrow{D A}=-\overrightarrow{B C}$
c. $\overrightarrow{A C} \& \overrightarrow{D B} ; \overrightarrow{A E} \& \overrightarrow{E B} ; \overrightarrow{E C} \& \overrightarrow{D E}$; $\overrightarrow{A B} \& \overrightarrow{C B}$
4. 


a. $\overrightarrow{A B}=\overrightarrow{C D}$
b. $\overrightarrow{A B}=-\overrightarrow{E F}$
c. $|\overrightarrow{A B}|=|\overrightarrow{E F}|$ but $\overrightarrow{A B} \neq \overrightarrow{E F}$
d. $\overrightarrow{G H}=2 \overrightarrow{A B}$
e. $\overrightarrow{A B}=-2 \overrightarrow{J I}$
8. a. $400 \mathrm{~km} / \mathrm{h}$, due south
b. $70 \mathrm{~km} / \mathrm{h}$, southwesterly
c. $30 \mathrm{~km} / \mathrm{h}$ southeasterly
d. $25 \mathrm{~km} / \mathrm{h}$, due east
9. a. i. False; they have equal magnitude, but opposite direction.
ii. True; they have equal magnitude.
iii. True; the base has sides of equal length, so the vectors have equal magnitude.
iv. True; they have equal magnitude and direction.
b. $\begin{aligned}|\overrightarrow{B D}| & =\sqrt{18},|\overrightarrow{B E}|=\sqrt{73}, \\ |\overrightarrow{B H}| & =\sqrt{82}\end{aligned}$

## Problems:

3. An airplane's direction is $\mathrm{E} 25^{\circ} \mathrm{N}$. Explain why this is the same as $\mathrm{N} 65^{\circ} \mathrm{E}$ or a bearing of $65^{\circ}$.
4. In the discussion, we defined $\frac{1 \vec{x} \text {. Using your own scale, draw your own }}{\hat{x}}$ vector to represent $\vec{x}$.
a. Sketch $\frac{1}{x} \vec{x}$ and describe this vector in your own words.
b. Sketch $-\frac{1}{r} \vec{x}$ and describe this vector in your own words.

## Answers:

3. $E 25^{\circ} \mathrm{N}$ describes a direction that is $25^{\circ}$ toward the north of due east. $\mathrm{N} 65^{\circ} \mathrm{E}$ and "a bearing of 65 " both describe a direction that is $65^{\circ}$ toward the east of due north.
4. a. $\frac{1}{|\vec{x}|} \vec{x}$ is a vector with length 1 unit in the same direction as $\vec{x}$.
b. $-\frac{1}{|\vec{x}|} \vec{x}$ is a vector with length 1 unit in the opposite direction of $\vec{x}$.

## Problems:



## 6. ABCDEFGH is a rectangular prism.

a. Wite a single vector that is equivalent to $\overrightarrow{E G}+\overrightarrow{G H}+\overrightarrow{H D}+\overrightarrow{D C}$.
b. Write a vector that is equivalent to $\overrightarrow{E G}+\overrightarrow{G D}+\overrightarrow{D E}$.
c. Is it true that $|\overrightarrow{H B}|=|\overrightarrow{G A}|$ ? Explain.
7. Write the following vector in simplified form:

$$
3(\vec{a}-2 \vec{b}-5 \vec{c})-3(2 \vec{a}-4 \vec{b}+2 \vec{c})-(\vec{a}-3 \vec{b}+3 \vec{c})
$$

11. A cube is constructed from the three vectors $\vec{a}, \vec{b}$, and $\vec{c}$, as shown below.

a. Express each of the diagonals $\overrightarrow{A G}, \overrightarrow{B H}, \overrightarrow{C E}$, and $\overrightarrow{D F}$ in terms of $\vec{a}, \vec{b}$, and $\vec{c}$.
b. Is $|\overrightarrow{A G}|=|\overrightarrow{B H}|$ ? Explain.

## Answers:

6. a. $\overrightarrow{E C}$
b. $\overrightarrow{0}$
c. Yes, the diagonals of a rectangular prism are of equal length.
7. $-4 \vec{a}+9 \vec{b}-24 \vec{c}$
8. a. $\overrightarrow{A G}=\vec{a}+\vec{b}+\vec{c}$, $\overrightarrow{B H}=-\vec{a}+\vec{b}+\vec{c}$,
$\overrightarrow{C E}=-\vec{a}-\vec{b}+\vec{c}$, $\overrightarrow{D F}=\vec{a}-\vec{b}+\vec{c}$
b. $|\overrightarrow{A G}|^{2}=|\vec{a}|^{2}+|\vec{b}|^{2}+|\vec{c}|^{2}$

$$
=|-\vec{a}|^{2}+|\vec{b}|^{2}+|\vec{c}|^{2}
$$

$$
=\mid \overrightarrow{B H}^{2}
$$

$$
\therefore|\overrightarrow{A G}|=|\overrightarrow{B H}|
$$

