Displacement, Velocity and Acceleration Date: _____

If $f(t) = s$ then	also called s(t)
$f'(t) = v$ and	also called v(t)
$f''(t) = a$	also called a(t)

When stating answers to motion questions, you should always interpret the signs of *s*, *v*, and *a*. All questions have a point of reference O, usually called the origin.

Consider this:

A particle moves **along the y axis** with this relationship between position and time:

$$s(t) = t^3 - 8t^2 + 17t - 10$$
$$= (t-1)(t-2)(t-5)$$

Therefore, we have:

$$v(t) = 3t^2 - 16t + 17 \quad \text{and} \quad a(t) = 6t - 16$$
$$\approx 3(t-1.5)(t-3.9)$$

1) Sketch the graph of s(t).

Location of the particle:

1) When is the particle:

   a) above the x axis?
   b) below the x-axis?
   c) In general, when does this occur for any $s(t)$? What is needed to answer this question?

Movement of the particle:

1) When is the particle:

   a) not moving?
   b) turning around?
   c) In general, when does this occur for any $s(t)$? What is needed to answer this question?

2) When is the particle:

   a) moving up?
   b) moving down?
   c) stopped?
   d) In general, when does this occur for any $v(t)$? What is needed to answer this question?

3) When is the particle:

   a) moving towards the origin?
   b) moving away from the origin?
   c) In general, when does this occur for any $s(t)$ and $v(t)$? What is needed to answer this question?

Speed of the particle:

1) When is the particle:

   a) have constant speed?
   b) increasing its speed?
   c) decreasing its speed?
   d) In general, when does this occur for any $a(t)$? What is needed to answer this question?

For motion of a particle on the y axis:
(for motion on the x axis, substitute 'right' for 'up', 'left' for 'down'.)
For a given function $s(t)$, at any time t, the particle

may be *located*:	above the x axis……	$s > 0$
	below the x axis……	$s < 0$
	at a stationary point (not moving)……	$v = 0$
	at a turning point (turning around) (changing direction).	v changes sign
	at a maximum point…….	v changes from + to –
	at a minimum point……..	v changes from – to +
may be *moving*:	upward…….	$v > 0$
	downward……	$v < 0$
	stopped……	$v = 0$
	toward the origin…….	$sv < 0$
	away from the origin……	$sv > 0$
may have *speed* ($\lvert v \rvert$):	constant……	$a = 0$
	increasing (speeding up)……..	$av > 0$
	decreasing (slowing down)……..	$av < 0$

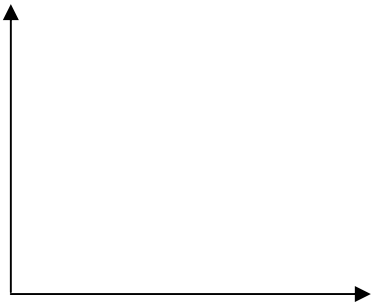
(all references above are to instantaneous velocities and accelerations)

Note: Some problems specify that $t > 0$. Don't assume this unless it is specifically stated.
When stating answers to motion questions, you should always interpret the signs of s, v, and a.
(Example if $s = -10$, $v = 5$, and $a = -3$: the particle is located 10 units below the origin, moving at 5 units/second upward, slowing down at a rate of 3 units/ second^2.)
All questions have a point of reference O, usually called the origin.

Ramp it Up

A ball was rolled **up** a ramp away from a motion detector (CBR) and was allowed to roll back down the ramp towards the motion detector. The data was collected in a graphing calculator with time in seconds in List 1 and distance in meters in List 2.

1. Graph the data using the graphing calculator and sketch your results.



WINDOW:
Xmin=
Xmax=
Xscl=
Ymin=
Ymax=
Yscl=

2. Using graphing technology, determine an algebraic model for the data.

3. Determine the initial distance of the ball from the CBR using the graphing calculator and your model from above.

4. Find the time(s) at which the ball is 4.0 m from the CBR using the graphing calculator and your model from above.

5. Find the average speed of the ball from $t = 0.5$ seconds to $t = 1.5$ seconds.

6. Describe the motion of the ball at $t = 3$ seconds using the graphing calculator and calculus. (where it is, the direction its moving and if it slowing down or speeding up)

7. Determine the time when the instantaneous speed is 0.6 m/s using the graphing calculator and calculus.

8. Determine the time when the ball is farthest from the CBR using the graphing calculator and calculus. Find the farthest distance the ball gets from the CBR using the graphing calculator and calculus.

9. Determine the total distance travelled in the first three seconds using the graphing calculator and algebra.

10. Using the graphing calculator and calculus, determine the time interval(s) during which the ball is:

    a) moving up the ramp.
    b) speeding up

Ramp it down, up and then down again

A ball was rolled **down** a ramp towards a motion detector (CBR), then the ramp was changed so that the ball could roll away from the CBR. Finally the ramp was returned to its original position and the ball rolled back down towards the motion detector. The data was collected in a graphing calculator with time in seconds in List 3 and distance in meters in List 4.

1. Graph the data using the graphing calculator and sketch your results.

WINDOW:
Xmin=
Xmax=
Xscl=
Ymin=
Ymax=
Yscl=

2. Using graphing technology, determine an algebraic model for the data.

3. Determine the initial distance of the ball from the CBR using the graphing calculator and your model from above.

4. Find the time(s) at which the ball is 4.0 m from the CBR using the graphing calculator and your model from above.

5. Find the average speed of the ball from $t = 0.5$ seconds to $t = 1.5$ seconds.

6. Describe the motion of the ball at $t = 3$ seconds using the graphing calculator and calculus. (where it is, the direction its moving and if it slowing down or speeding up)

7. Determine the time when the instantaneous speed is 0.6 m/s using the graphing calculator and calculus.

8. Determine the time when the ball is farthest from the CBR using the graphing calculator and calculus. Find the farthest distance the ball gets from the CBR using the graphing calculator and calculus.

9. Determine the total distance travelled in the first five seconds using the graphing calculator and algebra.

10. Using the graphing calculator and calculus, determine the time interval(s) during which the ball is:

    c) Moving towards the CBR.
    d) speeding up

Displacement, Velocity and Acceleration Word Problems

1. A particle moves on the y axis with this relationship between position and time:
$s(t) = t^3 - 14t^2 + 49t - 36$.

Determine the time interval(s) during which it is :

a) located above the origin — $1 < t < 4$ *or* $t > 9$

b) moving downward — $\frac{7}{3} < t < 7$

c) speeding up — $\frac{7}{3} < t < \frac{14}{3}$ *or* $t > 7$

d) moving towards the origin
$t < 1$ *or* $\frac{7}{3} < t < 4$ *or* $7 < t < 9$

2. A particle moves on the y axis with this relationship between position and time:
$s(t) = t^3 - 17t^2 + 80t - 100$.

Determine the time interval(s) during which it is :

a) located below the origin — $t < 2$ *or* $5 < t < 10$

b) moving upward — $t < \frac{10}{3}$ *or* $t > 8$

c) slowing down — $t < \frac{10}{3}$ *or* $\frac{17}{3} < t < 8$

d) moving away from the origin — $2 < t < \frac{10}{3}$ *or* $5 < t < 8$ *or* $t > 10$

3. A particle moves on the y axis with this relationship between position and time:
$$s(t) = \frac{1}{4}t^4 - 2t^3 + \frac{9}{2}t^2 - 4t + 2$$

a) Describe the motion of the particle at $t = 0$. — Located 2 units above origin, moving down at $4\ ^{units}/_{sec}$ and slowing down at $9\ ^{units}/_{sec^2}$.

b) What is the average velocity of the particle between $t = 1$ and $t = 4$ seconds? — $2.25\ ^{units}/_{sec}\downarrow$

c) When does the particle reverse direction? — $t = 4$

d) Find the total distance traveled from $t = 0$ to $t = 5$ seconds. — 14.75 units

4. A particle moves on the y axis with this relationship between position and time:
$$s(t) = \frac{1}{4}t^4 - \frac{2}{3}t^3 - \frac{5}{2}t^2 + 6t + 4,\ t \in \Re$$

a) Describe the motion of the particle at $t = 0$.

b) What is the average velocity of the particle between $t = 1$ and $t = 3$ seconds? — $-2.7\ ^{units}/_{sec}$

c) When does the particle reverse direction? — $t = -2, 1, 3$ seconds

d) Find the total distance traveled from $t = 0$ to $t = 3$ seconds. — 8.5 units

5. A particle moves on the y axis with this relationship between position and time:

$s(t) = 4t^3 - 4t^2 - 15t, \quad t \in \Re$

a) Determine the time interval(s) during which the particle is moving towards the origin.

$$t < \frac{-3}{2} \text{ or } \frac{-5}{6} < t < 0 \text{ or } \frac{3}{2} < t < \frac{5}{2}$$

b) Determine the time interval(s) during which the particle is speeding up.

$$\frac{-5}{6} < t < \frac{1}{3} \text{ or } t > \frac{3}{2}$$

c) Find the total distance traveled between $t = 1$ and $t = 3$ seconds. 48 units

7. A particle moves on the y axis with this relationship between distance and time:

$$s(t) = \frac{1}{4}t^4 - \frac{5}{3}t^3 + \frac{7}{2}t^2 - 3t + 12$$

a) Describe the motion of the particle when $t = 0$. Located 12 units above origin, moving down at 3 $^{units}/_{sec}$ and slowing down at $7\ ^{units}/_{sec^2}$.

b) What is the average velocity between $t = 1$ and $t = 4$ seconds? $^{3}/_{4}\ ^{units}/_{sec}$

c) When does the particle reverse direction? 3 seconds

d) Find the total distance travelled between $t = 0$ and $t = 4$ seconds? 5.83 units

8. A particle moves on the y axis so that its displacement at any time t ($t \in \Re$) is given by: $s(t) = t^3 - 5t^2 - 8t + 12$.

Determine the time interval(s) during which it is:

a) located below the origin $t < -2 \text{ or } 1 < t < 6$

b) moving upwards $t < \frac{-2}{3} \text{ or } t > 4$

c) speeding up $\frac{-2}{3} < t < \frac{5}{3} \text{ or } t > 4$

d) moving away from the origin $-2 < t < \frac{-2}{3} \text{ or } 1 < t < 4 \text{ or } t > 6$

9. A particle moves on the y axis with this relationship between distance and time:

$$s(t) = \frac{1}{4}t^4 - 4t^3 + \frac{39}{2}t^2 - 28t + 11$$

a) What is the displacement, velocity and acceleration of the particle when $t = 0$?
Located 11 units above origin, moving down at 28 $^{units}/_{sec}$ and slowing down at $39\ ^{units}/_{sec^2}$.

b) What is the average velocity between $t = 1$ and $t = 3$ seconds? $8\ ^{units}/_{sec}$

c) When does the particle reverse direction? $t = 1,4,7$ seconds

d) Find the total distance travelled between $t = 1$ and $t = 5$ seconds? 24.5 units

Text: Section 3.1 Pages 127-129 # 1, 3-12, 14
Section 3.2 Pages 135-138 # 5
Mid Chapter Review Page140 # 8-11
Section 5.1 Page 233 # 13